

thm_2Ealist_2EALL__DISTINCT__fmap__to__alist__keys (TMN15br9sk7LZ5gfJND1tmvTcenXX9Htgb6)

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Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \tag{4}$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{5}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 4 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (6)$$

Definition 5 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2E$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (7)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (8)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in (A_27a^{(2^{A_27a})}) \quad (9)$$

Definition 6 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2z \in 2.V2z))))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap (c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 15 We define $c_2Epred_set_2E_DIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 16 We define $c_2Epred_set_2E_DELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (ap$

Definition 17 We define $c_2Epred_set_2E_REST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (ap (c_2Epred_set_2E$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (13)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (14)$$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in$

Definition 20 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2^{A_27a}$

Definition 21 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x) \wedge$

Definition 22 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})^{A_27a}$

Definition 23 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A_27a$

Definition 24 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 25 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21 (2^{A_27a}$

Definition 26 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (A_27a^{A_27a}$

Definition 27 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in (A_27a^{A_27a}$

Definition 28 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}$

Definition 29 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}$

Definition 30 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}$

Definition 31 We define $c_2Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota.(ap (ap (c_2Erelation_2EWFREC (2^{A-27a}$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A-27a})}) \end{aligned} \quad (15)$$

Definition 32 We define $c_2Ealist_2Efmap_to_alist$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Efinite_map$

Definition 33 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2ET)$.

Definition 34 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 35 We define $c_2Epred_set_2Ecompl$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Epred_set$

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT \\ & A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(2^{A-27a})})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Definition 36 We define $c_2Efinite_map_2Efdomsub$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0fm \in (ty_2Efinite_m$

Definition 37 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A-27c}).\lambda V1$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (17)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (18)$$

Definition 38 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))) \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ (\forall V5y_{.27} \in A_{.27a}.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_{.27a} \\ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_{.27a} \ V1Q) \ V3x_{.27} \\ V5y_{.27})))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ (ap \ V0f \ V1v)))) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in A_{.27a}.((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_{.27a} \ c_2Ebool_2ET) \ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \ (ap \ (c_2Ebool_2ECOND \ A_{.27a} \ c_2Ebool_2EF) \ V2t1) \ V3t2) = V3t2)))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0v \in 2.((p \ (ap \ c_2Ebool_2EBOUNDED \ V0v)) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty \ A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}). \\ (\forall V2x \in A_{.27c}.((ap \ (ap \ (ap \ (c_2Ecombin_2Eo \ A_{.27c} \ A_{.27b} \ A_{.27a}) \\ V0f) \ V1g) \ V2x) = (ap \ V0f \ (ap \ V1g \ V2x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0fm \in (ty_2Efinite_map_2Efmap\ A.27a\ A.27b).(p\ (ap\ (c.2Epred_set_2EFINITE \\ & \quad A.27a)\ (ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0fm)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0fm \in (ty_2Efinite_map_2Efmap\ A.27a\ A.27b).(\forall V1k \in \\ & A.27a.((ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map_2Efdomsub \\ & \quad A.27a\ A.27b)\ V0fm)\ V1k)) = (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27a) \\ & \quad (ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0fm))\ V1k)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0f \in (A.27b^{A.27a}).((ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b) \\ & \quad V0f)\ (c.2Elist_2ENIL\ A.27a)) = (c.2Elist_2ENIL\ A.27b))) \wedge (\forall V1f \in \\ & \quad (A.27b^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty_2Elist_2Elist \\ & \quad A.27a).((ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ (ap\ (ap\ (c.2Elist_2ECONS \\ & \quad \quad A.27a)\ V2h)\ V3t)) = (ap\ (ap\ (c.2Elist_2ECONS\ A.27b)\ (ap\ V1f\ V2h)) \\ & \quad \quad (ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b)\ V1f)\ V3t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f1 \in (A.27b^{A.27a}).(\forall V1f2 \in (A.27b^{A.27a}).(\forall V2l \in \\ & \quad (ty_2Elist_2Elist\ A.27a).(((ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b) \\ & \quad \quad V0f1)\ V2l) = (ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b)\ V1f2)\ V2l)) \Leftrightarrow (\forall V3e \in \\ & \quad A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V3e)\ (ap\ (c.2Elist_2ELIST_TO_SET \\ & \quad \quad A.27a)\ V2l)))) \Rightarrow ((ap\ V0f1\ V3e) = (ap\ V1f2\ V3e)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}).(\forall V1g \in (A.27b^{A.27a}). \\ & \quad (\forall V2l \in (ty_2Elist_2Elist\ A.27a).((ap\ (ap\ (c.2Elist_2EMAP \\ & \quad \quad A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27b)\ V1g)\ V2l)) = \\ & \quad \quad (ap\ (ap\ (c.2Elist_2EMAP\ A.27a\ A.27c)\ (ap\ (ap\ (c.2Ecombin_2Eo\ A.27a \\ & \quad \quad \quad A.27c\ A.27b)\ V0f)\ V1g))\ V2l)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l \in (ty_2Elist_2Elist\ A_27a).(\forall V1f \in (A_27b^{A_27a}). \\
& \quad (\forall V2x \in A_27b.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2x)\ (ap\ (\\
& \quad c_2Elist_2ELIST_TO_SET\ A_27b)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a \\
& \quad A_27b)\ V1f)\ V0l)))) \Leftrightarrow (\exists V3y \in A_27a.((V2x = (ap\ V1f\ V3y)) \wedge (\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3y)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27a)\ V0l)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\
& \quad A_27a)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True) \wedge (\forall V0h \in A_27a.(\\
& \quad \forall V1t \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A_27a)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a) \\
& \quad V1t)))) \wedge (p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ V1t)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((p\ (ap \\
& \quad (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow ((ap\ (c_2Elist_2ESET_TO_LIST \\
& \quad A_27a)\ V0s) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Elist_2Elist\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})\ V0s)\ (c_2Epred_set_2EEMPTY \\
& \quad A_27a))))\ (c_2Elist_2ENIL\ A_27a))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
& \quad (ap\ (c_2Epred_set_2ECHOICE\ A_27a)\ V0s))\ (ap\ (c_2Elist_2ESET_TO_LIST \\
& \quad A_27a)\ (ap\ (c_2Epred_set_2EREST\ A_27a)\ V0s)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\\
& \quad \forall V1s \in (2^{A_27a}).(((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\
& \quad V1s)) \wedge (\neg(V1s = (c_2Epred_set_2EEMPTY\ A_27a)))) \Rightarrow (p\ (ap\ V0P\ (ap \\
& \quad (c_2Epred_set_2EREST\ A_27a)\ V1s)))) \Rightarrow (p\ (ap\ V0P\ V1s)))) \Rightarrow (\forall V2v \in \\
& \quad (2^{A_27a}).(p\ (ap\ V0P\ V2v))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Elist_2ESET_TO_LIST \\
& \quad A_27a)\ (c_2Epred_set_2EEMPTY\ A_27a)) = (c_2Elist_2ENIL\ A_27a))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).((p\ (ap \\ (c.2Epred_set_2EFINITE\ A.27a)\ V0s)) \Rightarrow (\forall V1x \in A.27a.((\\ p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V1x)\ (ap\ (c.2Elist_2ELIST_TO_SET \\ A.27a)\ (ap\ (c.2Elist_2ESET_TO_LIST\ A.27a)\ V0s)))) \Leftrightarrow (p\ (ap\ (ap \\ (c.2Ebool_2EIN\ A.27a)\ V1x)\ V0s)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ A.27b.(((ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair_2EFST\ A.27a \\ A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0P \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}).((\exists V1p \in \\ (ty_2Epair_2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p_1 \in \\ A.27a.(\exists V3p_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair_2E_2C \\ A.27a\ A.27b)\ V2p_1)\ V3p_2)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1x \in \\ A.27a.(\forall V2y \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V1x) \\ (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\ (c.2Ebool_2EIN\ A.27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\neg(p\ (ap \\ (ap\ (c.2Ebool_2EIN\ A.27a)\ (ap\ (c.2Epred_set_2ECHOICE\ A.27a) \\ V0s))\ (ap\ (c.2Epred_set_2EREST\ A.27a)\ V0s)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in \\ (2^{A.27a}).((p\ (ap\ (c.2Epred_set_2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set_2EDELETE \\ A.27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (c.2Epred_set_2EFINITE\ A.27a)\ V1s)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap \\ & (c_2Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_{.27a})\ (ap\ (c_2Epred_set_2EREST\ A_{.27a})\ V0s)))))) \end{aligned} \quad (57)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0fm \in (ty_2Efinite_map_2Efm\ A_{.27a}\ A_{.27b}).(p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ & A_{.27a})\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}) \\ & A_{.27a})\ (c_2Epair_2EFST\ A_{.27a}\ A_{.27b}))\ (ap\ (c_2Ealist_2Efm_to_alist \\ & A_{.27a}\ A_{.27b})\ V0fm)))))) \end{aligned}$$