

thm_2Ealist_2EALOOKUP__APPEND (TMTto- BGGfro1f2K3Bmnp2GMRnZceNB53mae6)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (2)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (3)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Let $c_2Ealist_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Ealist_2EALOOKUP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{A_27b})^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a))}) \quad (5)$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (6)$$

Definition 10 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (7)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (8)$$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (9)$$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS\ A_27a) ($

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (10)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0v \in A_27b. (\forall V1s \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad \quad A_27a\ A_27b)). (\forall V2k \in A_27a. (\forall V3ls2 \in (ty_2Elist_2Elist \\
& \quad \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)). (((ap\ (ap\ (c_2Ealist_2EALOOKUP \\
& \quad \quad \quad A_27b\ A_27a)\ V1s)\ V2k) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V0v)) \Rightarrow (\\
& \quad \quad (ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad \quad \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ V1s)\ V3ls2))\ V2k) = (ap\ (c_2Eoption_2ESOME \\
& \quad \quad \quad A_27b)\ V0v))) \wedge (((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ V1s) \\
& \quad \quad V2k) = (c_2Eoption_2ENONE\ A_27b)) \Rightarrow ((ap\ (ap\ (c_2Ealist_2EALOOKUP \\
& \quad \quad \quad A_27b\ A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ (ty_2Epair_2Eprod\ A_27a \\
& \quad \quad \quad A_27b))\ V1s)\ V3ls2))\ V2k) = (ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b \\
& \quad \quad \quad A_27a)\ V3ls2)\ V2k))))))))) \\
& \hspace{10em} (11)
\end{aligned}$$

Assume the following.

$$True \hspace{10em} (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \hspace{2em} (13)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\
& \quad (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& \quad \quad p\ V0t)))))) \\
& \hspace{10em} (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& \quad A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\
& \quad \quad (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \\
& \hspace{10em} (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad \quad \quad A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a)\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad \quad A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& \quad \quad \quad (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& \quad \quad \quad A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \\
& \hspace{10em} (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& \quad A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& \quad \quad \quad A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{10em} (17)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0l1 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & \quad (\forall V1l2 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & \quad (\forall V2k \in A_27a.((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a) \\ & \quad (ap\ (ap\ (c_2Elist_2EAPPEND\ (ty_2Epair_2Eprod\ A_27a\ A_27b))\ V0l1) \\ V1l2))\ V2k) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27b\ (ty_2Eoption_2Eoption \\ A_27b))\ (ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ V0l1)\ V2k))\ (\\ ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ V1l2)\ V2k))\ (\lambda V3v \in \\ A_27b.(ap\ (c_2Eoption_2ESOME\ A_27b)\ V3v)))))) \end{aligned}$$