

thm_2Ealist_2EALOOKUP__ZIP__MAP__SND
(TMXHJwScyJVSiqT8bGPLLP64Cir.JziNAK9d)

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Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (3)$$

Let $c_2Ealist_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Ealist_2EALOOKUP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{A_27b})^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a))}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap\ (c_2Ebool_2E_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_2E_21\ 2)\ (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (a$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (5)$$

Definition 10 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 11 We define c_2Epair_2E2C to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Let $c_2Elist_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EZIP \\ & A_27a\ A_27b \in ((ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))^{(ty_2Epair_2Eprod\ (ty_2Elist_2Elist\ A_27a\ A_27b))}) \end{aligned} \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (13)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (14)$$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (15)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (16)$$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (17)$$

Definition 16 We define $c_2Eoption_2EENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0e)$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P)\ V0P)))$

Definition 18 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 19 We define $c_2Eoption_2EESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2EOPTION_MAP\ A_27b)^{(ty_2Eoption_2EOPTION_MAP\ A_27a)})^{(A_27b)^{A_27a}}) \quad (18)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (20)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ V0m)$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0q \in A.27b.((ap\ (ap\ (c.2Ealist.2EALOOKUP\ A.27a\ A.27b) \\
& (c.2Elist.2ENIL\ (ty.2Epair.2Eprod\ A.27b\ A.27a)))\ V0q) = (c.2Eoption.2ENONE \\
& A.27a))) \wedge (\forall V1y \in A.27a.(\forall V2x \in A.27b.(\forall V3t \in \\
& (ty.2Elist.2Elist\ (ty.2Epair.2Eprod\ A.27b\ A.27a)).(\forall V4q \in \\
& A.27b.((ap\ (ap\ (c.2Ealist.2EALOOKUP\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Elist.2ECONS \\
& (ty.2Epair.2Eprod\ A.27b\ A.27a))\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b \\
& A.27a)\ V2x)\ V1y))\ V3t))\ V4q) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ (ap\ (ap\ (c.2Emin.2E.3D\ A.27b)\ V2x)\ V4q))\ (ap\ (c.2Eoption.2ESOME \\
& A.27a)\ V1y))\ (ap\ (ap\ (c.2Ealist.2EALOOKUP\ A.27a\ A.27b)\ V3t)\ V4q))))))))) \\
& \hspace{15em} (21)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \hspace{2em} (23)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \hspace{15em} (24)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \hspace{15em} (25)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \hspace{2em} (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \hspace{2em} (27)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \hspace{2em} (28)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \hspace{2em} (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\ ap\ V0P\ V1a)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\
& (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\
& \forall V1t \in (ty_2Elist_2Elist\ A_27a).((ap\ (c_2Elist_2ELENGTH \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\
& (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((V0l = (c_2Elist_2ENIL\ A_27a)) \vee (\exists V1h \in A_27a. (\\
& \exists V2t \in (ty_2Elist_2Elist\ A_27a).(V0l = (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V1h)\ V2t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a).((c_2Enum_2E0 = (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l)) \Leftrightarrow (\\
& V0l = (c_2Elist_2ENIL\ A_27a))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (((ap\ (c.2Elist.2EZIP \\
& A.27c\ A.27d)\ (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Elist.2Elist\ A.27c) \\
& (ty.2Elist.2Elist\ A.27d))\ (c.2Elist.2ENIL\ A.27c))\ (c.2Elist.2ENIL \\
& A.27d))) = (c.2Elist.2ENIL\ (ty.2Epair.2Eprod\ A.27c\ A.27d))) \wedge \\
& (\forall V0x1 \in A.27a. (\forall V1l1 \in (ty.2Elist.2Elist\ A.27a). \\
& (\forall V2x2 \in A.27b. (\forall V3l2 \in (ty.2Elist.2Elist\ A.27b). \\
& ((ap\ (c.2Elist.2EZIP\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Elist.2Elist \\
& A.27a)\ (ty.2Elist.2Elist\ A.27b))\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a) \\
& V0x1)\ V1l1))\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ V2x2)\ V3l2)))) = (ap \\
& (ap\ (c.2Elist.2ECONS\ (ty.2Epair.2Eprod\ A.27a\ A.27b))\ (ap\ (ap\ (\\
& c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x1)\ V2x2))\ (ap\ (c.2Elist.2EZIP\ A.27a \\
& A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ (ty.2Elist.2Elist\ A.27a)\ (ty.2Elist.2Elist \\
& A.27b))\ V1l1)\ V3l2))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum. (\neg((ap\ c.2Enum.2ESUC\ V0n) = c.2Enum.2E0))) \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. (((ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption.2ESOME \\
& A.27a)\ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0f \in (A.27b^{A.27a}). (\forall V1x \in A.27a. ((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27a\ A.27b)\ V0f)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (ap\ (c.2Eoption.2ESOME \\
& A.27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A.27b^{A.27a}). ((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27a\ A.27b)\ V2f)\ (c.2Eoption.2ENONE\ A.27a)) = (c.2Eoption.2ENONE \\
& A.27b))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27a). (\forall V2y \in A.27b. (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A.27a. ((V1x = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& V3z))))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \\
& \tag{49}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0l1 \in (\\
& \quad ty_2Elist_2Elist\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27b). \\
& \quad (\forall V2k \in A_27c. (\forall V3f \in (A_27d^{A_27b}). (((ap\ (c_2Elist_2ELENGTH \\
A_27a)\ V0l1) = (ap\ (c_2Elist_2ELENGTH\ A_27b)\ V1l2)) \Rightarrow ((ap\ (c_2Ealist_2EALOOKUP \\
A_27d\ A_27a)\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27d)\ (ap\ (ap\ (c_2Epair_2E_2C \\
(ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27d))\ V0l1)\ (ap \\
(ap\ (c_2Elist_2EMAP\ A_27b\ A_27d)\ V3f)\ V1l2)))))) = (ap\ (ap\ (c_2Ecombin_2Eo \\
A_27a\ (ty_2Eoption_2Eoption\ A_27d)\ (ty_2Eoption_2Eoption\ A_27b)) \\
(ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27d)\ V3f))\ (ap\ (c_2Ealist_2EALOOKUP \\
A_27b\ A_27a)\ (ap\ (c_2Elist_2EZIP\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C \\
(ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2))))))))))
\end{aligned}$$