

thm_2Ealist_2EFLOOKUP__FUPDATE__LIST__ALOOKUP__SOM
(TMcu2d5SchioMjEgyHZ2ra89nHYKwTXW4K3)

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Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (3)$$

Let $c_2Ealist_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Ealist_2EALOOKUP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{A_27b})(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a))) \quad (4)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \quad (7)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP \\ & A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (8)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (9)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))))$

Definition 4 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFDOM\ A_27a\ A_27b)$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (10)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in (A_27a^{(2^{A_27a})}) \quad (11)$$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 8 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 10 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 11 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (13)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_7E\ V0t)\ c_2Ebool_2E_7E\ V0t)$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E\ V0t))$

Definition 14 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Ebool_2E_7E\ V0t)\ c_2Ebool_2E_7E\ V0t)$

Definition 15 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (c_2Ebool_2E_7E\ V0t)\ c_2Ebool_2E_7E\ V0t)$

Definition 16 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (ap\ (c_2Epred_set_2EDELETE\ V0s)\ c_2Ebool_2E_7E\ V0t))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist \\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \end{aligned} \quad (14)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist \\ A_27a) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A)\ p)$ **of type** $\iota \Rightarrow \iota$.

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 19 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ V0t)\ c_2Ebool_2E_21\ V0t)$

Definition 20 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 21 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 22 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a))\ c_2Ebool_2E_21\ V0t))$

Definition 23 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0s)\ c_2Ebool_2E_7E\ V0t)))$

Definition 24 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E_21\ V0t)\ c_2Ebool_2E_21\ V0t)$

Definition 25 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (2^{A_27a}).(ap\ (ap\ (c_2Erelation_2EWF\ V0f)\ c_2Ebool_2E_21\ V0t))$

Definition 26 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.(\lambda V3c \in A_27a.(\lambda V4d \in A_27a.))$

Definition 27 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (2^{A_27a}).(ap\ (ap\ (c_2Erelation_2ETC\ V0R)\ c_2Ebool_2E_21\ V0t))$

Definition 28 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 29 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 30 We define $c_2Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota. (ap (ap (c_2Erelation_2EWFREC (2^{A_27a})$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Definition 31 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (17)$$

Definition 32 We define $c_2Ealist_2Efmap_to_alist$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s \in (ty_2Efinite_map$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist \\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (18)$$

Definition 33 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40\ ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (19)$$

Definition 34 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS \\ A_27a\ A_27b \in ((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a}}) \end{aligned} \quad (20)$$

Definition 35 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE \\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2E}} \end{aligned} \quad (21)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (23)$$

Definition 36 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR \\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27a})}) \end{aligned} \quad (24)$$

Definition 37 We define $c_2Ealist_2Ealist_to_fmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Elist_2Elist\ A_27a\ A_27b)$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27a})}) \end{aligned} \quad (25)$$

Definition 38 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Elist_2Elist\ A_27a\ A_27b))$

Let $c_2Efinite_map_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUNION \\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (26)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (27)$$

Definition 39 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a))$

Definition 40 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS\ A_27a\ A_27b))$

Definition 41 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a))$

Definition 42 We define $c_2Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b)$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (28)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (29)$$

Definition 43 We define $c_2Eprod_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Eprod_set_2EUNION\ A_27a\ V0s\ V1t))$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0fm \in (ty_2Efinite_map_2Efm\ A_27a\ A_27b). ((ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ (ap\ (c_2Ealist_2Efm_to_alist\ A_27a\ A_27b)\ V0fm)) = V0fm)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). (\forall V1k \in A_27a. (\forall V2v \in A_27b. (((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ V0al)\ V1k) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2v)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1k)\ V2v))\ (ap\ (c_2Elist_2ELIST_TO_SET\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ V0al))))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). (\forall V1k \in A_27a. (\forall V2v \in A_27b. (((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27b\ A_27a)\ V0al)\ V1k) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2v)) \Rightarrow (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ V0al))\ V1k) = V2v)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). ((ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ V0al)) = (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0al)))))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l1 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\
& \quad (\forall V1l2 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\
& \quad ((ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ V0l1)\ V1l2)) = (ap\ (ap\ (c_2Efinite_map_2EFUNION \\
& \quad A_27a\ A_27b)\ (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ V0l1)) \\
& \quad (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ V1l2)))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0ls \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\
& \quad (\forall V1fm \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b).((ap\ (ap \\
& \quad (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V1fm)\ V0ls) = (\\
& \quad ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (c_2Elist_2EREVERSE\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b))\ V0ls))\ (ap\ (c_2Ealist_2Efmap_to_alist\ A_27a\ A_27b) \\
& \quad V1fm)))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \hspace{2em} (37)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \hspace{15em} (38)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \hspace{15em} (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \hspace{2em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\
& (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \hspace{2em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\
& ap\ V0P\ V1a))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (\forall V1g \in \\
& (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (((ap\ (c_2Efinite_map_2EFDOM \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27a\ A_27b)\ V0f) \\
& V1g)) = (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& A_27a\ A_27b)\ V0f))\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V1g)))) \wedge \\
& (\forall V2x \in A_27a. ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a \\
& A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27a\ A_27b)\ V0f)\ V1g)) \\
& V2x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V0f)))\ (ap \\
& (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ V0f)\ V2x))\ (ap\ (ap\ (\\
& c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ V1g)\ V2x)))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0l \in (ty_2Elist_2Elist\ A_27a). (\forall V1f \in (A_27b^{A_27a}). \\
& (\forall V2x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2x)\ (ap\ (\\
& c_2Elist_2ELIST_TO_SET\ A_27b)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a \\
& A_27b)\ V1f)\ V0l)))) \Leftrightarrow (\exists V3y \in A_27a. ((V2x = (ap\ V1f\ V3y)) \wedge (\\
& p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3y)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& A_27a)\ V0l)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a). ((ap\ (c_2Elist_2EREVERSE\ A_27a)\ (ap\ (c_2Elist_2EREVERSE \\
& A_27a)\ V0l)) = V0l))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption.2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A.27a. \\ & (\forall V2y \in A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\ & A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\ & A.27a)) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (\\ & ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a))\ V0P)\ (c.2Eoption.2ENONE \\ & A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (c.2Eoption.2ENONE \\ & A.27a)) \Leftrightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\ & A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\ & A.27a)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\ & (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a)) \\ & V0P)\ (c.2Eoption.2ENONE\ A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a) \\ & V1x)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\ & V2y)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair.2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{(ty.2Epair.2Eprod\ A.27a\ A.27b)}). ((\exists V1p \in \\ & (ty.2Epair.2Eprod\ A.27a\ A.27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p_{-1} \in \\ & A.27a. (\exists V3p_{-2} \in A.27b. (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair.2E_2C \\ & A.27a\ A.27b)\ V2p_{-1})\ V3p_{-2})))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ & (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred._set.2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2.((p \vee 0A) \Rightarrow ((\neg(p \vee 0A)) \Rightarrow \text{False}))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \vee 0A) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \vee 0A)) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \vee 0A)) \Rightarrow \text{False}) \Rightarrow (((p \vee 0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Leftrightarrow (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \vee (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Rightarrow (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (p \vee 0p)))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \vee 0p) \Rightarrow (p \vee 1q))) \Rightarrow (\neg(p \vee 1q)))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (74)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0ls \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a)). \\ & \quad (\forall V1k \in A_27b.(\forall V2v \in A_27a.(\forall V3fm \in (ty_2Efinite_map_2Efmap \\ & \quad A_27b\ A_27a).(((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27a\ A_27b)\ V0ls) \\ V1k) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2v)) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFLOOKUP \\ & \quad A_27b\ A_27a)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27b\ A_27a) \\ & \quad V3fm)\ (ap\ (c_2Elist_2EREVERSE\ (ty_2Epair_2Eprod\ A_27b\ A_27a)) \\ & \quad V0ls)))\ V1k) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2v))))))) \end{aligned}$$