

thm_2Ealist_2EFUPDATE__LIST__EQ__APPEND__REVERSE
(TMND-
JnY6mngjpEptVWnGMJDpbuEbrt7WVdW)

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Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)) \tag{4}$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \tag{5}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x))$

Definition 4 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2E$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (7)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in (A_27a^{(2^{A_27a})}) \quad (8)$$

Definition 5 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2z \in 2.V2z)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)))$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 14 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 15 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (ap$

Definition 16 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (ap (c_2Epred_set_2E$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (12)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (13)$$

Definition 17 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$).

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda$

Definition 19 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2$

Definition 20 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 21 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 22 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Definition 23 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E40$

Definition 24 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E21$

Definition 25 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M$

Definition 26 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 27 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 28 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 29 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 30 We define $c_2Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota.(ap (ap (c_2Erelation_2EWFREC (2^{A_27a})$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 31 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFAPPLY)$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (15)$$

Definition 32 We define $c_2Ealist_2Efmmap_to_alist$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Efinite_map_2EFMAP_TO_ALIST)$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (17)$$

Definition 33 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (18)$$

Definition 34 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum))$

Let $c_2Efinite_map_2Efmmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmmap_ABS \\ A_27a\ A_27b \in ((ty_2Efinite_map_2Efmmap\ A_27a\ A_27b)^{((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})}) \end{aligned} \quad (19)$$

Definition 35 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Efinite_map_2Efmmap_ABS))$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE \\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2Efmmap_ABS)}) \end{aligned} \quad (20)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (21)$$

Definition 36 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR \\ & A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \end{aligned} \quad (22)$$

Definition 37 We define $c_2Ealist_2Ealist_to_fmap$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s \in (ty_2Elist_2Elist)$

Let $c_2Efinite_map_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUNION \\ & A_27a\ A_27b \in (((ty_2Efinite_map_2E fmap\ A_27a\ A_27b)^{(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)})^{(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)}) \end{aligned} \quad (23)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ & A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \end{aligned} \quad (24)$$

Definition 38 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap\ (c_2Elist_2Elist))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (25)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0k \in A_27c. \\ & (\forall V1v \in A_27d. (\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\ & A_27c\ A_27d)). ((ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b) \\ & (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))) = (c_2Efinite_map_2EFEMPTY \\ & A_27a\ A_27b)) \wedge ((ap\ (c_2Ealist_2Ealist_to_fmap\ A_27c\ A_27d) \\ & (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27c\ A_27d))\ (ap\ (\\ & ap\ (c_2Epair_2E_2C\ A_27c\ A_27d)\ V0k)\ V1v))\ V2t)) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\ & A_27c\ A_27d)\ (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27c\ A_27d)\ V2t)) \\ & (ap\ (ap\ (c_2Epair_2E_2C\ A_27c\ A_27d)\ V0k)\ V1v)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0fm \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).((ap\ (c_2Ealist_2Ealist_to_fmap \\ & \quad A.27a\ A.27b)\ (ap\ (c_2Ealist_2E fmap_to_alist\ A.27a\ A.27b)\ V0fm)) = \\ & \quad V0fm)) \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A.27a\ A.27b)). \\ & ((ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (c_2Ealist_2Ealist_to_fmap \\ & \quad A.27a\ A.27b)\ V0al)) = (ap\ (c_2Elist_2ELIST_TO_SET\ A.27a)\ (ap \\ & (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A.27a\ A.27b)\ A.27a)\ (c_2Epair_2EFST \\ & \quad A.27a\ A.27b))\ V0al)))) \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0l1 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A.27a\ A.27b)). \\ & \quad (\forall V1l2 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A.27a\ A.27b)). \\ & ((ap\ (c_2Ealist_2Ealist_to_fmap\ A.27a\ A.27b)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\ & (ty_2Epair_2Eprod\ A.27a\ A.27b))\ V0l1)\ V1l2)) = (ap\ (ap\ (c_2Efinite_map_2EFUNION \\ & \quad A.27a\ A.27b)\ (ap\ (c_2Ealist_2Ealist_to_fmap\ A.27a\ A.27b)\ V0l1)) \\ & \quad (ap\ (c_2Ealist_2Ealist_to_fmap\ A.27a\ A.27b)\ V1l2)))))) \end{aligned} \tag{30}$$

Assume the following.

$$True \tag{31}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{34}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{35}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\
& (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0g \in (ty_2Efinite_map_2Efmap A_27a A_27b). ((ap (ap (\\
& c_2Efinite_map_2EFUNION A_27a A_27b) (c_2Efinite_map_2EFEMPTY \\
& A_27a A_27b)) V0g) = V0g))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0f \in (ty_2Efinite_map_2Efmap A_27a A_27b). ((ap (ap (\\
& c_2Efinite_map_2EFUNION A_27a A_27b) V0f) (c_2Efinite_map_2EFEMPTY \\
& A_27a A_27b)) = V0f))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1g \in \\
& \quad (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V2x \in A.27a.(\\
& \quad \forall V3y \in A.27b.((ap\ (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b) \\
& (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& A.27a\ A.27b)\ V2x)\ V3y)))\ V1g) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b)\ V0f) \\
& V1g))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2x)\ V3y)))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1g \in \\
& \quad (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V2x \in A.27a.(\\
& \quad \forall V3y \in A.27b.((ap\ (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b) \\
& V0f)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V1g)\ (ap\ (\\
& ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2x)\ V3y))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& (ty_2Efinite_map_2E fmap\ A.27a\ A.27b))\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A.27a)\ V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0f)))\ (ap \\
& (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b)\ V0f)\ V1g))\ (ap\ (ap\ (\\
& c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION \\
& A.27a\ A.27b)\ V0f)\ V1g))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V2x) \\
& V3y)))))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(((ap\ (ap \\
& (c_2Efinite_map_2EFUPDATE_LIST\ A.27a\ A.27b)\ V0f)\ (c_2Elist_2ENIL \\
& (ty_2Epair_2Eprod\ A.27a\ A.27b))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod \\
& A.27a\ A.27b).(\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& A.27a\ A.27b)).((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A.27a \\
& A.27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A.27a\ A.27b)) \\
& V1h)\ V2t)) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A.27a\ A.27b) \\
& (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V0f)\ V1h))\ V2t)))))) \\
& \hspace{15em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A.27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A.27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A.27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (((ap\ (c.2Elist.2EREVERSE\ A.27a) \\
& \quad (c.2Elist.2ENIL\ A.27a)) = (c.2Elist.2ENIL\ A.27a)) \wedge (\forall V0h \in \\
& \quad A.27a. (\forall V1t \in (ty.2Elist.2Elist\ A.27a). ((ap\ (c.2Elist.2EREVERSE \\
& \quad A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t)) = (ap\ (ap\ (c.2Elist.2EAPPEND \\
& \quad A.27a)\ (ap\ (c.2Elist.2EREVERSE\ A.27a)\ V1t))\ (ap\ (ap\ (c.2Elist.2ECONS \\
& \quad A.27a)\ V0h)\ (c.2Elist.2ENIL\ A.27a))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in (ty.2Epair.2Eprod\ A.27a\ A.27b). (\exists V1q \in A.27a. \\
& \quad (\exists V2r \in A.27b. (V0x = (ap\ (ap\ (c.2Epair.2E_2C\ A.27a\ A.27b) \\
& \quad \quad V1q)\ V2r))))))
\end{aligned} \tag{49}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0ls \in (ty.2Elist.2Elist\ (ty.2Epair.2Eprod\ A.27a\ A.27b)). \\
& \quad (\forall V1fm \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). ((ap\ (ap \\
& \quad (c.2Efinite_map.2EFUPDATE_LIST\ A.27a\ A.27b)\ V1fm)\ V0ls) = (\\
& \quad ap\ (c.2Ealist.2Ealist_to_fmap\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Elist.2EAPPEND \\
& \quad (ty.2Epair.2Eprod\ A.27a\ A.27b))\ (ap\ (c.2Elist.2EREVERSE\ (ty.2Epair.2Eprod \\
& \quad A.27a\ A.27b))\ V0ls))\ (ap\ (c.2Ealist.2Efmap_to_alist\ A.27a\ A.27b) \\
& \quad \quad V1fm))))))
\end{aligned}$$