thm_2Ealist_2EPERM__fmap__to__alist (TMMehP4Z25Vfy8nJSxJufAc8rpzYnZe44ju)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (1)$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (2)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (3)$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ (c$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \quad (6)$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \quad (7)$$

Let $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (8)$$

Definition 12 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_ma$

Definition 13 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (9)$$

Definition 15 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

Definition 16 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Efinite__map_2EFLOOKUP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{10}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{11}$$

Let $c\_2Ealist\_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ealist\_2EALOOKUP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))}) \tag{12}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_ABS\ A\_27a\ A\_27b \in ((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})}) \tag{13}$$

Definition 19 *We define* c_2Efinite__map_2EFEMPTY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Efinite\_\_map\_2E$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{14}$$

Definition 20 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efinite\_\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2EFUPDAT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_\_map} \tag{15}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{16}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{17}$$

Definition 21 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27b})^{A\_27a})}) \quad (18)$$

Definition 22 *We define* c_2Ealist_2Ealist__to__fmap *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (ty\_2Elist\_2Elist\ ($

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (19)$$

Let $c\_2Epred\_\_set\_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \quad (20)$$

Definition 23 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 24 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (21)$$

Definition 25 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 26 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 27 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 28 *We define* c_2Epred__set_2EREST *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (ap$ (c_2Epred__set_2E

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (22)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (23)$$

Definition 29 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 30 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 31 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 32 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 33 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 34 *We define* c_2Erelation_2EWF *to be* $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ ($c_2Ebool_2E_21

Definition 35 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 36 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 37 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 38 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 39 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 40 *We define* c_2Elist_2ESET__TO__LIST *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Erelation_2EWFREC $(2^{A\_}$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{24}$$

Definition 41 *We define* c_2Ealist_2Efmap__to__alist *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (ty\_2Efinite\_\_map$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{25}$$

Let $c\_2Elist\_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{26}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{27}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{28}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{29}$$

Definition 42 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 43 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Elist\_2EALL\_\_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (30)$$

Definition 44 *We define* $\mathsf{c\_2Emarker\_2EAbbrev}$ *to be* $\lambda V0x \in 2.V0x$.

Definition 45 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNIV}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Definition 46 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 47 *We define* $\mathsf{c\_2Epred\_\_set\_2EINJ}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (31)$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (32)$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (33)$$

Definition 48 *We define* $\mathsf{c\_2Esorting\_2EPERM}$ *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0al \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).(\forall V1fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27c\ A\_27d).(((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Ealist\_2Ealist\_\_to\_\_fmap}\ A\_27a\ A\_27b)\ V0al)) = (ap\ (c\_2Ealist\_2EALOOKUP\ A\_27b\ A\_27a)\ V0al)) \wedge ((ap\ (c\_2Ealist\_2EALOOKUP\ A\_27d\ A\_27c)\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27c\ A\_27d)\ V1fm)) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27c\ A\_27d)\ V1fm))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\forall V1fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0p)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V1fm)))) \Leftrightarrow ((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ V1fm)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0p)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0p)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((ap\ (\mathsf{c\_2Ealist\_2Ealist\_\_to\_\_fmap}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V0fm)) = V0fm)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0al \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)). (\forall V1k \in A\_27a.(\forall V2v \in A\_27b.(((ap\ (ap\ (c\_2Ealist\_2EALOOKUP\ A\_27b\ A\_27a)\ V0al)\ V1k) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V2v)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1k)\ V2v))\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0al))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0al \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)). ((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Ealist\_2Ealist\_\_to\_\_fmap}\ A\_27a\ A\_27b)\ V0al)) = (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27a)\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b))\ V0al)))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27a)\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V0fm))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1f2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(((ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V0f1) = (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V1f2)) \Rightarrow (V0f1 = V1f2)))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0fm1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1fm2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c). (((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V0fm1) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27c)\ V1fm2)) \Rightarrow ((ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27a)\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V0fm1)) = (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ A\_27a)\ (c\_2Epair\_2EFST\ A\_27a\ A\_27c))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27c)\ V1fm2)))))) \tag{41}$$

Assume the following.

$$True \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{45}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{52}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{53}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \tag{54}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (c\_2Elist\_2ELENGTH\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ V0l)) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l))) \Rightarrow (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Elist\_2EEL\ A\_27b)\ V0n)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V2f)\ V1l)) = (ap\ V2f\ (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V0n)\ V1l))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l))) \Leftrightarrow (\exists V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2n)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \land (V1x = (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V2n)\ V0l)))))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V0l)) \Leftrightarrow (\forall V1n1 \in ty\_2Enum\_2Enum.(\forall V2n2 \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n1)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l))) \land (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2n2)\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l)))) \Rightarrow (((ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V1n1)\ V0l) = (ap\ (ap\ (c\_2Elist\_2EEL\ A\_27a)\ V2n2)\ V0l)) \Leftrightarrow (V1n1 = V2n2))))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l1))\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V2l2)))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27b))) \land ((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ V1l1) = (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ V2l2))) \Rightarrow (V1l1 = V2l2))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \land (V1y = V3b))))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \land (V1y = V3b))))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{83}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0L \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0L)\ V0L))) \tag{84}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))\Rightarrow((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l2))))) \tag{85}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))\Rightarrow((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l2))))) \tag{86}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1)\ V2l2))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ V2l2))))))) \tag{87}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))\Rightarrow(\forall V2a \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2a)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l1)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2a)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l2)))))))) \tag{88}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0fm1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1fm2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V0fm1))\ (ap\ (\mathsf{c\_2Ealist\_2Efmap\_\_to\_\_alist}\ A\_27a\ A\_27b)\ V1fm2)))\Leftrightarrow(V0fm1 = V1fm2))))$$