

thm_2Ealist_2Ealist__to__fmap__PERM (TMJCeRjLFk3Eqqc3iyymmGuLHrqSpXLuv2sq)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (V0P)) (V0P)))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2E fmap A0 A1) \quad (4)$$

Let $c_2Efinite_map_2E fmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (5)$$

Definition 11 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap (c_2Efinite_map_2E$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2E$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (10)$$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (11)$$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR \\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{(A_27b^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 14 We define $c_2Ealist_2Ealist_to_fmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Elist_2Elist$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP \\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (13)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 15 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (15)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in \\ (A_27a^{(2^{A_27a})}) \end{aligned} \quad (16)$$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (17)$$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2E$

Definition 20 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 21 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (ap$

Definition 22 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (ap\ (c_2Epred_set_2E$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (18)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (19)$$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 24 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E21\ 2$

Definition 25 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 26 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 27 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 28 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E40$

Definition 29 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2E21$

Definition 30 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1$

Definition 31 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b$

Definition 32 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 33 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 34 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 35 We define $c_2Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Erelation_2EWFREC\ (2^{A_27a})$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (20)$$

Definition 36 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \quad (21)$$

Definition 37 We define $c_2Ealist_2E fmap_to_alist$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Efinite_map$
Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in \quad (22)$$

$$(2^{(ty_2Elist_2Elist\ A_27a)})$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (23)$$

Definition 38 We define $c_2Esorting_2E PERM$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist\ A_27a).\lambda V1L2$
Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\quad (24)$$

$$\forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)).$$

$$((p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP$$

$$(ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b)))$$

$$V0al))) \Rightarrow (p\ (ap\ (ap\ (c_2Esorting_2E PERM\ (ty_2Epair_2Eprod\ A_27a$$

$$A_27b))\ (ap\ (c_2Ealist_2E fmap_to_alist\ A_27a\ A_27b)\ (ap\ (c_2Ealist_2Ealist_to_ fmap$$

$$A_27a\ A_27b)\ V0al)))\ V0al))))$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\quad (25)$$

$$\forall V0fm1 \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b).(\forall V1fm2 \in$$

$$(ty_2Efinite_map_2E fmap\ A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Esorting_2E PERM$$

$$(ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (c_2Ealist_2E fmap_to_alist$$

$$A_27a\ A_27b)\ V0fm1))\ (ap\ (c_2Ealist_2E fmap_to_alist\ A_27a\ A_27b)$$

$$V1fm2))) \Leftrightarrow (V0fm1 = V1fm2))))$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow$$

$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (($$

$$(p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (27)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge$$

$$((\neg False) \Leftrightarrow True))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$

$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg($$

$$p\ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\ A_27a). (\forall V1y \in (ty_2Elist_2Elist \ A_27a). (\forall V2z \in \\ (ty_2Elist_2Elist \ A_27a). (((p (ap (ap (c_2Esorting_2EPERM \ A_27a) \\ V0x) \ V1y)) \wedge (p (ap (ap (c_2Esorting_2EPERM \ A_27a) \ V1y) \ V2z))) \Rightarrow (\\ p (ap (ap (c_2Esorting_2EPERM \ A_27a) \ V0x) \ V2z))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist \ A_27a). ((p (ap (ap (c_2Esorting_2EPERM \\ A_27a) \ V0l1) \ V1l2)) \Leftrightarrow (p (ap (ap (c_2Esorting_2EPERM \ A_27a) \ V1l2) \\ V0l1)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist \ A_27a). ((p (ap (ap (c_2Esorting_2EPERM \\ A_27a) \ V0l1) \ V1l2)) \Rightarrow ((p (ap (c_2Elist_2EALL_DISTINCT \ A_27a) \\ V0l1)) \Leftrightarrow (p (ap (c_2Elist_2EALL_DISTINCT \ A_27a) \ V1l2)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}). (\forall V1l1 \in (ty_2Elist_2Elist \ A_27a). \\ (\forall V2l2 \in (ty_2Elist_2Elist \ A_27a). ((p (ap (ap (c_2Esorting_2EPERM \\ A_27a) \ V1l1) \ V2l2)) \Rightarrow (p (ap (ap (c_2Esorting_2EPERM \ A_27b) (ap (\\ ap (c_2Elist_2EMAP \ A_27a \ A_27b) \ V0f) \ V1l1)) (ap (ap (c_2Elist_2EMAP \\ A_27a \ A_27b) \ V0f) \ V2l2)))))) \end{aligned} \quad (46)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0l1 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & \quad (\forall V1l2 \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\ & \quad ((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ (ty_2Epair_2Eprod\ A_27a\ A_27b)) \\ & \quad V0l1)\ V1l2)) \wedge (p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ (ap\ (ap\ (\\ & \quad c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27a)\ (c_2Epair_2EFST \\ & \quad A_27a\ A_27b))\ V0l1)))) \Rightarrow ((ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a \\ & \quad A_27b)\ V0l1) = (ap\ (c_2Ealist_2Ealist_to_fmap\ A_27a\ A_27b)\ V1l2)))) \end{aligned}$$