

thm_2Ealist_2Ealist__to__fmap__to__alist__PERM (TMN5rJeXZjQpuahyS1hp5JhDnoawHZ8PC42)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2 to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)) P))$

Definition 6 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E2F))$

Definition 9 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.V2t)) t1 t2))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone))$
Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (7)$$

Let $c_2Ealist_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Ealist_2EALOOKUP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27a)^{A_27b})^{(ty_2Elist_2Elist (ty_2Epair_2Eprod A_27b A_27a))}) \quad (8)$$

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2E fmap A0 A1) \quad (9)$$

Let $c_2Efinite_map_2E fmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (10)$$

Definition 12 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E fmap A_27a A_27b) (c_2Eone_2Eone))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod A_27a A_27b) (V0x V1y))$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE \\ & A_27a\ A_27b \in (((ty_2Efinite_map_2Efm\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ & A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (14)$$

Definition 14 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b})$

Let $c_2Elist_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDR \\ & A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27b})^{A_27a}} \end{aligned} \quad (15)$$

Definition 15 We define $c_2Ealist_2Ealist_to_fmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (ty_2Elist_2Elist\ A_27a)$

Let $c_2Efinite_map_2Efm_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efm_REP \\ & A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efm\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (17)$$

Definition 16 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2Efm\ A_27a\ A_27b)$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (18)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in \\ & (A_27a)^{(2^{A_27a})} \end{aligned} \quad (19)$$

Definition 17 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(20)

Definition 20 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V1s)$

Definition 21 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V1t)$

Definition 22 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap\ (ap\ (c_2Ebool_2E_21\ 2)\ V1x)\ V0s)$

Definition 23 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (ap\ (c_2Epred_set_2EDELETE)\ V0s))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})$$
(21)

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)$$
(22)

Definition 24 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Ebool_2E_21\ 2)\ V2t2))))$

Definition 25 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V0s)$

Definition 26 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 27 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c)^{A_27b})^{A_27a})$

Definition 28 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a)^{A_27a})\ A_27a))$

Definition 29 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40)\ V0P)))$

Definition 30 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V0R)$

Definition 31 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b)^{A_27a}. \lambda V1R \in (A_27b)^{A_27a}. (ap\ (ap\ (c_2Erelation_2EWF)\ V1R)\ V0f)$

Definition 32 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27a. (ap\ (ap\ (c_2Erelation_2ERESTRICT)\ V1a)\ V0R)$

Definition 33 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27b)^{A_27a}. (ap\ (ap\ (c_2Erelation_2ETC)\ V1M)\ V0R)$

Definition 34 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 35 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 36 We define $c_2Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota. (ap (ap (c_2Erelation_2EWFREC (2^{A_27a})$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \end{aligned} \quad (23)$$

Definition 37 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}}) \end{aligned} \quad (24)$$

Definition 38 We define $c_2Ealist_2Efmap_to_alist$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s \in (ty_2Efinite_map$

Definition 39 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b)^{A_27c}. \lambda V1M$

Definition 40 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Definition 41 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 42 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a)^{(ty_2Eoption_2Eoption\ A_27a)} \quad (25)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (26)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (27)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (28)$$

Definition 43 We define $c_Esorting_2EPERM$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist A_27a).\lambda V1L2$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0q \in A_27b.((ap (ap (c_2Ealist_2EALOOKUP A_27a A_27b) \\ & (c_2Elist_2ENIL (ty_2Epair_2Eprod A_27b A_27a))) V0q) = (c_2Eoption_2ENONE \\ & A_27a))) \wedge (\forall V1y \in A_27a.(\forall V2x \in A_27b.(\forall V3t \in \\ & (ty_2Elist_2Elist (ty_2Epair_2Eprod A_27b A_27a)).(\forall V4q \in \\ & A_27b.((ap (ap (c_2Ealist_2EALOOKUP A_27a A_27b) (ap (ap (c_2Elist_2ECONS \\ & (ty_2Epair_2Eprod A_27b A_27a)) (ap (ap (c_2Epair_2E_2C A_27b \\ & A_27a) V2x) V1y)) V3t)) V4q) = (ap (ap (ap (c_2Ebool_2ECOND (ty_2Eoption_2Eoption \\ & A_27a)) (ap (ap (c_2Emin_2E_3D A_27b) V2x) V4q)) (ap (c_2Eoption_2ESOME \\ & A_27a) V1y)) (ap (ap (c_2Ealist_2EALOOKUP A_27a A_27b) V3t) V4q))))))))) \\ & \hspace{15em} (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0al \in (ty_2Elist_2Elist (ty_2Epair_2Eprod A_27a A_27b)). \\ & ((ap (c_2Ealist_2Efmap_to_alist A_27a A_27b) (ap (c_2Ealist_2Ealist_to_fmap \\ & A_27a A_27b) V0al)) = (ap (ap (c_2Elist_2EMAP A_27a (ty_2Epair_2Eprod \\ & A_27a A_27b)) (\lambda V1k \in A_27a.(ap (ap (c_2Epair_2E_2C A_27a A_27b) \\ & V1k) (ap (c_2Eoption_2ETHE A_27b) (ap (ap (c_2Ealist_2EALOOKUP \\ & A_27b A_27a) V0al) V1k)))))) (ap (c_2Elist_2ESET_TO_LIST A_27a) \\ & (ap (c_2Elist_2ELIST_TO_SET A_27a) (ap (ap (c_2Elist_2EMAP \\ & (ty_2Epair_2Eprod A_27a A_27b) A_27a) (c_2Epair_2EFST A_27a A_27b)) \\ & V0al)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$True \hspace{15em} (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\ & \hspace{15em} (32) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \hspace{15em} (33)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \hspace{15em} (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & A_27a.(p V0t)) \Leftrightarrow (p V0t))) \\ & \hspace{15em} (35) \end{aligned}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p V0t)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow \text{False}))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \vee (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \text{False}) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p V0t)))))) \quad (41)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge ((\neg \text{False}) \Leftrightarrow \text{True}))) \quad (42)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (43)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t)))))) \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\ A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in \\ A.27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (47)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\exists V1x \in \\ A.27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). (((p\ V0P) \wedge (\forall V2x \in A.27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ A.27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (49)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). (((p\ V0P) \vee (\exists V2x \in A.27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \quad (50)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A.27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (51)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A.27a}). ((\forall V2x \in A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a. (p\ (ap\ V1Q\ V3x)))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (55)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & (\\ \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. & \\ (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) & \\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f & \\ V2x))\ (ap\ V0f\ V3y)))))) & \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in & \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow & \\ ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) & \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. & \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. & \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge & \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) & \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) & \\ V5y_27)))))) & \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & (\\ \forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in & \\ A_27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}). & \\ \forall V4x \in A_27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) & \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in & \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) & \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap & \\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) & \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\
& (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\
& V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI \\
A_27a)\ V0x) = V0x)) \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a.(\forall V1a1 \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V2a0.27 \in A_27a.(\forall V3a1.27 \in \\
& (ty_2Elist_2Elist\ A_27a).(((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\
& V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0.27)\ V3a1.27)) \Leftrightarrow ((V0a0 = \\
& V2a0.27) \wedge (V1a1 = V3a1.27))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0l \in (ty_2Elist_2Elist\ A_27a).(\forall V1f \in (A_27b^{A_27a}). \\
& (((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V0l) = (c_2Elist_2ENIL \\
& A_27b)) \Leftrightarrow (V0l = (c_2Elist_2ENIL\ A_27a))) \wedge (((c_2Elist_2ENIL\ A_27b) = \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V0l)) \Leftrightarrow (V0l = (c_2Elist_2ENIL \\
& A_27a))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f1 \in (A_27b^{A_27a}). (\forall V1f2 \in (A_27b^{A_27a}). (\forall V2l \in \\
& \quad (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f1)\ V2l) = (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f2)\ V2l)) \Leftrightarrow (\forall V3e \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3e)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27a)\ V2l))) \Rightarrow ((ap\ V0f1\ V3e) = (ap\ V1f2\ V3e))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}). (\forall V1g \in (A_27b^{A_27a}). \\
& \quad (\forall V2l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EMAP \\
& \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1g)\ V2l)) = \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a \\
& \quad A_27c\ A_27b)\ V0f)\ V1g))\ V2l))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0l \in (ty_2Elist_2Elist\ A_27a). (\forall V1f \in (A_27b^{A_27a}). \\
& \quad (\forall V2x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V2x)\ (ap\ (\\
& \quad c_2Elist_2ELIST_TO_SET\ A_27b)\ (ap\ (ap\ (c_2Elist_2EMAP\ A_27a \\
& \quad A_27b)\ V1f)\ V0l)))) \Leftrightarrow (\exists V3y \in A_27a. ((V2x = (ap\ V1f\ V3y)) \wedge (\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3y)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27a)\ V0l))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\
& \quad A_27a)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True) \wedge (\forall V0h \in A_27a. (\\
& \quad \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A_27a)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a) \\
& \quad V1t)))) \wedge (p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ V1t))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Eoption_2ETHE \\
A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x)) \tag{72}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a. \\ & (\exists V2r \in A_27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (75)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (76)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (79)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (80)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & (p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0L \in (ty_2Elist_2Elist \\
& A_27a). (p (ap (ap (c_2Esorting_2EPERM A_27a) V0L) V0L)))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& A_27a). (\forall V1y \in (ty_2Elist_2Elist A_27a). (\forall V2z \in \\
& (ty_2Elist_2Elist A_27a). (((p (ap (ap (c_2Esorting_2EPERM A_27a) \\
& V0x) V1y)) \wedge (p (ap (ap (c_2Esorting_2EPERM A_27a) V1y) V2z))) \Rightarrow (\\
& p (ap (ap (c_2Esorting_2EPERM A_27a) V0x) V2z))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& A_27a). (\forall V1l2 \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_2Esorting_2EPERM \\
& A_27a) V0l1) V1l2)) \Leftrightarrow (p (ap (ap (c_2Esorting_2EPERM A_27a) V1l2) \\
& V0l1))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0ls \in (ty_2Elist_2Elist \\
& A_27a). ((p (ap (c_2Elist_2EALL_DISTINCT A_27a) V0ls)) \Leftrightarrow (p (ap \\
& (ap (c_2Esorting_2EPERM A_27a) V0ls) (ap (c_2Elist_2ESET_TO_LIST \\
& A_27a) (ap (c_2Elist_2ELIST_TO_SET A_27a) V0ls))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1l1 \in (ty_2Elist_2Elist A_27a). \\
& (\forall V2l2 \in (ty_2Elist_2Elist A_27a). ((p (ap (ap (c_2Esorting_2EPERM \\
& A_27a) V1l1) V2l2)) \Rightarrow (p (ap (ap (c_2Esorting_2EPERM A_27b) (ap (\\
& ap (c_2Elist_2EMAP A_27a A_27b) V0f) V1l1)) (ap (ap (c_2Elist_2EMAP \\
& A_27a A_27b) V0f) V2l2))))))
\end{aligned} \tag{90}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0al \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})). \\ & \quad ((p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_{.27a})\ (ap\ (ap\ (c_2Elist_2EMAP \\ & \quad (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})\ A_{.27a})\ (c_2Epair_2EFST\ A_{.27a}\ A_{.27b})) \\ & \quad V0al))) \Rightarrow (p\ (ap\ (ap\ (c_2Esorting_2Eperm\ (ty_2Epair_2Eprod\ A_{.27a} \\ & \quad A_{.27b})\ (ap\ (c_2Elist_2Emap_to_alist\ A_{.27a}\ A_{.27b})\ (ap\ (c_2Elist_2Elist_to_fmap \\ & \quad A_{.27a}\ A_{.27b})\ V0al))))\ V0al)))) \end{aligned}$$