

thm\_2Ealist\_2Efmap\_\_to\_\_alist\_\_inj  
(TMUkxdv25YTvKTTigKkGMnpQW14uMCocvbW)

October 26, 2020

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Efinite\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let  $c\_2Efinite\_map\_2Efmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_REP\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)} \tag{4}$$

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{5}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 4** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (6)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ebool\_2EARB\ A.27a \in A.27a \quad (7)$$

Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Epred\_set\_2ECHOICE\ A.27a \in (A.27a^{(2^{A.27a})}) \quad (8)$$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c\_2Ebool\_2EF)$ .

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A.27a}).(ap\ V1f\ V0x)))$

**Definition 8** We define  $c\_2Emin\_2E.3D.3D.3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E.5C.2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 10** We define  $c\_2Ebool\_2E.2F.5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EABS\_prod\ A.27a\ A.27b \in ((ty\_2Epair\_2Eprod\ A.27a\ A.27b)^{(2^{A.27b})^{A.27a}}) \quad (10)$$

**Definition 11** We define  $c\_2Epair\_2E.2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap\ (c\_2Emin\_2E.3D.3D.3E\ V0t)\ (c\_2Ebool\_2E.7E\ V1t))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A.27a\ A.27b \in ((2^{A.27a})^{(ty\_2Epair\_2Eprod\ A.27a\ 2)^{A.27b}}) \quad (11)$$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A.27a}).(ap\ (c\_2Emin\_2E.3D.3D.3E\ V0t)\ (c\_2Ebool\_2E.7E\ V1s))$

**Definition 13** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E.3D.3D.3E\ V0t)\ c\_2Ebool\_2E.2F.5C\ V0t))$

**Definition 14** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap\ (c\_2Emin\_2E.3D.3D.3E\ V0s)\ (c\_2Ebool\_2E.7E\ V1t))$

**Definition 15** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1x \in A\_27a.(ap (ap$

**Definition 16** We define  $c\_2Epred\_set\_2EREST$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (ap (c\_2Epred\_set\_2E$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (12)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (13)$$

**Definition 17** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 19** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21 (2$

**Definition 20** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 21** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A-27b})^{A-27$

**Definition 22** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A-27a}) A$

**Definition 23** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 24** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 25** We define  $c\_2Erelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V1M$

**Definition 26** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 27** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1M$

**Definition 28** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1M$

**Definition 29** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1M$

**Definition 30** We define  $c\_2Elist\_2ESET\_TO\_LIST$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Erelation\_2EWFREC (2^{A-27$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EOUTL A\_27a A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum A\_27a A\_27b)}) \quad (14)$$

**Definition 31** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2EFAPPLY)$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP \\ & A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \end{aligned} \quad (15)$$

**Definition 32** We define  $c\_2Ealist\_2Efmmap\_to\_alist$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (ty\_2Efinite\_map\_2Ealist\_2Efmmap\_to\_alist)$

**Definition 33** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone))\ (\lambda V0x \in ty\_2Eone\_2Eone)$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \\ & A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (16)$$

**Definition 34** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_sum))$

Let  $c\_2Efinite\_map\_2Efmmap\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2Efmmap\_ABS \\ & A\_27a\ A\_27b \in ((ty\_2Efinite\_map\_2Efmmap\ A\_27a\ A\_27b)^{(ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a}}) \end{aligned} \quad (17)$$

**Definition 35** We define  $c\_2Efinite\_map\_2EFEMPTY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Efinite\_map\_2EFEMPTY))$

Let  $c\_2Efinite\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EFUPDATE \\ & A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2Efmmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_map\_2EFUPDATE)}) \end{aligned} \quad (18)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ & A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (19)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ & A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (20)$$

**Definition 36** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27b})$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDR \\ & A\_27a\ A\_27b \in (((A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b)^{A\_27a})^{A\_27a}} \end{aligned} \quad (21)$$

**Definition 37** We define  $c\_2Ealist\_2Ealist\_to\_fmap$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0s \in (ty\_2Elist\_2Elist$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (22)$$

**Definition 38** We define  $c\_2Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow (\forall V0fm \in (ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b). ((ap\ (c\_2Ealist\_2Ealist\_to\_fmap\ A\_27a\ A\_27b)\ (ap\ (c\_2Ealist\_2Efmap\_to\_alist\ A\_27a\ A\_27b)\ V0fm)) = V0fm)) \quad (23)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow (\forall V0al \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)). ((ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ (ap\ (c\_2Ealist\_2Ealist\_to\_fmap\ A\_27a\ A\_27b)\ V0al)) = (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ A\_27a)\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b))\ V0al)))) \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (26)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t) \Leftrightarrow (p\ V1x))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))) \quad (29)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (34)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (ty\_2Efinite\_map\_2Efmmap A\_27a A\_27b).(\forall V1g \in \\ & \quad (ty\_2Efinite\_map\_2Efmmap A\_27a A\_27b).(((ap (c\_2Efinite\_map\_2EFDOM \\ & \quad A\_27a A\_27b) V0f) = (ap (c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V1g)) \wedge \\ & \quad (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap ( \\ & \quad c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V0f))) \Rightarrow ((ap (ap (c\_2Efinite\_map\_2EFAPPLY \\ & \quad A\_27a A\_27b) V0f) V2x) = (ap (ap (c\_2Efinite\_map\_2EFAPPLY A\_27a \\ & \quad A\_27b) V1g) V2x)))))) \Leftrightarrow (V0f = V1g)) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2x \in A\_27b.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V2x)\ (ap\ ( \\
& \quad c\_2Elist\_2ELIST\_TO\_SET\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a \\
& \quad A\_27b)\ V1f)\ V0l)))) \Leftrightarrow (\exists V3y \in A\_27a.((V2x = (ap\ V1f\ V3y)) \wedge ( \\
& \quad p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3y)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ V0l)))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a \\
& \quad A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x)))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0P \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).((\exists V1p \in \\
& \quad (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p\_1 \in \\
& \quad A\_27a.(\exists V3p\_2 \in A\_27b.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad A\_27a\ A\_27b)\ V2p\_1)\ V3p\_2))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg( \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (52)$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f1 \in (ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b). (\forall V1f2 \in (ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b). (((ap\ (c\_2Ealist\_2Efm\_to\_alist\ A\_27a\ A\_27b)\ V0f1) = (ap\ (c\_2Ealist\_2Efm\_to\_alist\ A\_27a\ A\_27b)\ V1f2)) \Rightarrow (V0f1 = V1f2))))))$$