# thm_2Ealist_2Efupdate__list__funion (TML2RQggZGv2DfQMgjg7nWvdteF4WX76HLQ)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (3)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let $c\_2Ealist\_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ealist\_2EALOOKUP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a))}) \quad (5)$$

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (6)$$

**Definition 5** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 6** *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

**Definition 7** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 8** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

**Definition 9** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{7}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{8}$$

**Definition 10** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{9}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_ABS\ A\_27a\ A\_27b \in ((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})}) \tag{10}$$

**Definition 11** *We define* c_2Efinite__map_2EFEMPTY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Efinite\_\_map\_2E$

**Definition 12** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Efinite\_\_map\_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2EFUNI\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)})^{(ty\_2Efin} \tag{11}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{12}$$

**Definition 13** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ (c$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \tag{13}$$

Let $c\_2Esum\_2EOUTL : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{14}$$

**Definition 14** *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_ma$

**Definition 15** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

**Definition 16** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Esum\_2EISL : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{15}$$

**Definition 17** *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

**Definition 18** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 19** *We define* c_2Efinite__map_2EFLOOKUP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_m$

Let $c\_2Efinite\_\_map\_2EFUPDATE : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Efinite\_\_map\_2EFUPDAT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_\_map} \tag{16}$$

Let $c\_2Elist\_2EFOLDL : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{17}$$

**Definition 20** *We define* c_2Efinite__map_2EFUPDATE__LIST *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Elist\_2EI$

Let $c\_2Elist\_2ECONS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{18}$$

Let $c\_2Elist\_2ENIL : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{19}$$

**Definition 21** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{20}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{21}$$

**Definition 22** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

**Definition 23** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0l \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)). (\forall V1k \in A\_27a.(\forall V2m \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ (ap\ ( ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V2m)\ V0l))\ V1k) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27b\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (c\_2Ealist\_2EALOOKUP\ A\_27b\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0l))\ V1k))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ V2m)\ V1k))\ (\lambda V3v \in A\_27b.(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V3v))))))) \tag{22}$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{27}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{28}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. ((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a. ((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow ( \forall V0g \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b). ((ap\ (ap\ ( c\_2Efinite\_\_map\_2EFUNION\ A\_27a\ A\_27b)\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27b))\ V0g) = V0g)) \tag{34}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow ( \forall V0k \in A\_27b. ((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27b\ A\_27a))\ V0k) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a).(\forall V1k1 \in A\_27b.(\forall V2v \in A\_27a.(\forall V3k2 \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27b\ A\_27a)\ V0fm)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27a)\ V1k1)\ V2v)))\ V3k2) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27b)\ V1k1)\ V3k2))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2v))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ V0fm)\ V3k2))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a).(\forall V1f2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27a).(\forall V2k \in A\_27b.( (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUNION\ A\_27b\ A\_27a)\ V0f1)\ V1f2))\ V2k) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ V0f1)\ V2k))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27b\ A\_27a)\ V1f2)\ V2k))\ (\lambda V3v \in A\_27a.(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3v))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1f2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((V0f1 = V1f2) \Leftrightarrow ((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ V0f1) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFLOOKUP}\ A\_27a\ A\_27b)\ V1f2))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0f)\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) = V0f) \wedge (\forall V1h \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\forall V2t \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V1h)\ V2t)) = (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ V1h))\ V2t)))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( (\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.((ap\ ( ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL\ A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V3e \in A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a. (V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( (\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{46}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0m \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1l \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0m)\ V1l) = (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUNION\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27b))\ V1l))\ V0m))))$$