

thm_2Earithmetic_2EABS__DIFF__SUC__LE (TMHEQSknzHn9zDgSCUqh9TAH7gVtCLi9jx)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Definition 14 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 15 We define $c_Earithmetic_2EABS_DIFF$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1m \in ty_2Enum_2Enum$

Let $c_Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_Enum_2EZERO_REP \in \omega \quad (6)$$

Definition 16 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\ & (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ (ap\ c_Enum_2ESUC\ V0n))\ (ap \\ & c_Enum_2ESUC\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ V0n) \\ & V1m)))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ c_Enum_2E0)\ V0n))) \quad (8)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ V0m)\ (ap\ c_Enum_2ESUC\ V0m)))) \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D \\ & V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ V1n)\ V2p))) \Rightarrow (p\ (\\ & ap\ (ap\ c_Earithmetic_2E_3C_3D\ V0m)\ V2p)))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_Earithmetic_2E_3C_3D\ V0m)\ V0m))) \quad (11)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EABS_DIFF V0n) c_2Enum_2E0) = V0n) \wedge ((ap (ap c_2Earithmetic_2EABS_DIFF c_2Enum_2E0) V0n) = V0n))) \quad (12)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EABS_DIFF (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC V1m)) = (ap (ap c_2Earithmetic_2EABS_DIFF V0n) V1m)))) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \wedge (p V1t2)) \Leftrightarrow ((p V1t2) \wedge (p V0t1)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (18)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n)))) \quad (19)$$

Theorem 1

$$(\forall V0x \in ty_2Enum_2Enum.(\forall V1z \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2EABS_DIFF V0x) (ap c_2Enum_2ESUC V1z))) (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2EABS_DIFF V0x) V1z))))))$$