

thm_2Earithmetic_2EADD__DIV__RWT (TMTkXBsBUf1n84Yd2nyGGR1ShCt6yaNBtdu)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E))$

Definition 9 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 11 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A \rightarrow 2^a}).(ap V0P (ap (c_Emin_2E_40 A a))))$

Definition 13 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 14 We define $c_Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_Ebool_2E_2ET)$.

Definition 15 We define $c_Emarker_2E Cong$ to be $\lambda V0x \in 2.V0x$.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (10)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (11)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2A (ap \\
& (ap c_2Earithmic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmic_2E_2B \\
& (ap (ap c_2Earithmic_2E_2A V0m) V2p)) (ap (ap c_2Earithmic_2E_2A \\
& \quad V1n) V2p))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2B V0m) \\
& V1n) = (ap (ap c_2Earithmic_2E_2B V0m) V2p)) \Leftrightarrow (V1n = V2p))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2B V0m) \\
& V2p) = (ap (ap c_2Earithmic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((V1k = (ap (\\
& ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2A (ap (ap c_2Earithmic_2E_2EDIV \\
& \quad V1k) V0n)) V0n)) (ap (ap c_2Earithmic_2E_2EMOD V1k) V0n))) \wedge (p (ap \\
& (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmic_2E_2EMOD V1k) V0n)) \\
& \quad V0n))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& \quad \forall V2q \in ty_2Enum_2Enum. ((\exists V3r \in ty_2Enum_2Enum. (\\
& (V1k = (ap (ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2A \\
& \quad V2q) V0n)) V3r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3r) V0n)))) \Rightarrow (\\
& (ap (ap c_2Earithmic_2E_2EDIV V1k) V0n) = V2q))))
\end{aligned} \tag{17}$$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \tag{20}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0v \in 2. ((p\ (ap\ c_2Ebool_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (23)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V0n)) \Rightarrow (\forall V1m \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2EMOD\ V1m)\ V0n) = c_2Enum_2E0) \vee ((ap\ (ap\ c_2Earithmetic_2EMOD\ V2p)\ V0n) = c_2Enum_2E0)) \Rightarrow ((ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1m)\ V2p))\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2EDIV\ V1m)\ V0n))\ (ap\ (ap\ c_2Earithmetic_2EDIV\ V2p)\ V0n))))))))))$$