thm_2Earithmetic_2EADD__MOD (TMFJ63tA2hLnGJh1zEHVR5xgMbymhKnn4MH)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27})$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

1

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (7)$$

Definition 12 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \textsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \textsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V1n)) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))$$
$$(8)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1a \in ty\_2Enum\_2Enum.($$
$$\forall V2b \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})$$
$$V0n)) \Rightarrow (((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1a))$$
$$V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V2b))\ V0n)) \Leftrightarrow$$
$$((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1a)\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD$$
$$V2b)\ V0n)))))))$$
$$(9)$$

Assume the following.

$$True \qquad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \qquad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \qquad (12)$$
$$(((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\qquad (13)$$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))$$

2

Assume the following.

$$(\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow} True)) \tag{14}$$

Assume the following.

$$(\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}(p\ V0t))\wedge(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t)))\wedge(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1){\Rightarrow}((p\ V1t2){\Rightarrow}(p\ V2t3))){\Leftrightarrow}(((p\ V0t1)\wedge(p\ V1t2)){\Rightarrow}(p\ V2t3)))))) \tag{17}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x){\Leftrightarrow}(p\ V1x\_27))\wedge((p\ V1x\_27){\Rightarrow}((p\ V2y){\Leftrightarrow}(p\ V3y\_27)))){\Rightarrow}(((p\ V0x){\Rightarrow}(p\ V2y)){\Leftrightarrow}((p\ V1x\_27){\Rightarrow}(p\ V3y\_27))))))) \tag{18}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)){\Rightarrow}(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))){\Rightarrow}(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{19}$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1a \in ty\_2Enum\_2Enum.(\forall V2b \in ty\_2Enum\_2Enum.(\forall V3p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)){\Rightarrow}(((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1a)\ V3p))\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2b)\ V3p))\ V0n)){\Leftrightarrow}((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1a)\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V2b)\ V0n)))))))) $$