

thm_2Earithmetic_2EADD__MODULUS (TMLeYbYsrAfhFp1fgv1m96Jxn1MFQtXW7Rs)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda \tau a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2EREP_num m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{7}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 7 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmic_2EBIT1\ V0n))$.

Definition 8 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21\ 2))$.

Definition 12 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Definition 13 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A.\lambda y.y \in A))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define c_2Ebool_2E3F to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^A)^{2^a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A\ P)))$.

Definition 15 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$.

Definition 16 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & ((ap\ (ap\ c_2Earithmic_2E2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmic_2E2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmic_2E2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmic_2E2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmic_2E2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmic_2E2B\ V0m)\ V1n))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & (ap\ (ap\ c_2Earithmic_2E2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmic_2E2B \\ & V1n)\ V0m)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1j \in ty_2Enum_2Enum. (\forall V2k \in \\
& ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2EMOD V1j) V0n)) (ap (ap c_2Earithmetic_2EMOD \\
& \quad V2k) V0n))) V0n) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1j) V2k)) V0n))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD \\
& (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) V0n) = (ap (ap c_2Earithmetic_2EMOD \\
& \quad V1k) V0n))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (((ap (ap c_2Earithmetic_2EDIV V0n) V0n) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge \\
& ((ap (ap c_2Earithmetic_2EMOD V0n) V0n) = c_2Enum_2E0))))
\end{aligned} \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& \quad A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& \quad p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (\\
& 2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in \\
& \quad A_27a. (p (ap V1P V3x))) \vee (p V0Q))))))
\end{aligned} \tag{19}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge ((\neg(p V1q)) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (34)$$

Theorem 1

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Enum_2Enum. \\ & ((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\ & (ap (ap c_2Earithmetic_2E_2B V1x) V0n)) V0n) = (ap (ap c_2Earithmetic_2EMOD \\ & V1x) V0n)))))) \wedge (\forall V2n \in ty_2Enum_2Enum.(\forall V3x \in ty_2Enum_2Enum. \\ & ((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V2n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\ & (ap (ap c_2Earithmetic_2E_2B V2n) V3x)) V2n) = (ap (ap c_2Earithmetic_2EMOD \\ & V3x) V2n)))))) \end{aligned}$$