

thm_2Earithmetic_2EDA

(TMXgh2nr2j4Tf15hepF2EaB6o2C5Qwmfhfb)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (3)$$

Let $c_2Enum_2EAABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EAABS_num m)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Definition 15 We define $c_2Earithmetic_2EZERO$ to be $c_2Enum_2E0.$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E0\ ($

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Assume the following.

$$\begin{aligned} & ((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = \\ & \quad (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge ((ap\ (\\ & \quad ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (\\ & \quad ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & \quad V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V1n)\ V0m)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))) \\
 \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & \quad V1n) V0m)))) \\
 \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad (\neg(V1n = c_2Enum_2E0)) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap \\
 & \quad (ap c_2Earithmetic_2E_2B V0m) V1n)))) \\
 \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
 & \quad (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
 & \quad (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
 & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
 & \quad (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
 & \quad ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
 & \quad (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
 & \quad V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
 & \quad V0m) V1n))))))) \\
 \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
 & \quad (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
 & \quad (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
 & \quad V1n) V2p)))))) \\
 \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
 & \quad (V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & \quad c_2Earithmetic_2EZERO))))))) \\
 \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}).((\exists V1n \in ty_2Enum_2Enum. \\
 & (p (ap V0P V1n))) \Rightarrow (\exists V2n \in ty_2Enum_2Enum.((p (ap V0P V2n)) \wedge \\
 & (\forall V3m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V3m) V2n)) \Rightarrow (\neg(p (ap V0P V3m)))))))))) \\
 & (17)
 \end{aligned}$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \\
 & (19)
 \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
 & (21)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge \\
 & ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \\
 & (22)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & ((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
 & ((\neg False) \Leftrightarrow True))) \\
 & (23)
 \end{aligned}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap c_2Eenum_2ESUC V0n) = c_2Eenum_2E0))) \quad (25)$$

Theorem 1

$$\begin{aligned}
 & (\forall V0k \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(
 & (p (ap (ap c_2Eprim_rec_2E_3C c_2Eenum_2E0) V1n)) \Rightarrow (\exists V2r \in \\
 & ty_2Enum_2Enum.(\exists V3q \in ty_2Enum_2Enum.((V0k = (ap (ap c_2Earithmetic_2E_2B \\
 & (ap (ap c_2Earithmetic_2E_2A V3q) V1n)) V2r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & V2r) V1n)))))))) \\
 &
 \end{aligned}$$