

# thm\_2Earithmetic\_2EDIV\_\_LESS\_\_EQ (TMK- Mic3bHEWu29LKkE28BFyZiDJe5JpHXVN)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A a))))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) P) (ap (c\_2Emin\_2E\_40 A a))))$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 8** We define  $c\_2Ebool\_2E\_F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_F))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 11** We define  $c\_Enum\_ESUC$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ c\_Enum\_EABS\_num$

**Definition 12** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

**Definition 13** We define  $c\_Earithmetic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

Let  $c\_Enum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_EZERO\_REP \in \omega \tag{5}$$

**Definition 14** We define  $c\_Enum\_E0$  to be  $(ap\ c\_Enum\_EABS\_num\ c\_Enum\_EZERO\_REP)$ .

**Definition 15** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Enum\_E0$ .

Let  $c\_Earithmetic\_E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2B \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{6}$$

**Definition 16** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_Earithmetic$

**Definition 17** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{7}$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{8}$$

Let  $c\_Earithmetic\_E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2A \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum. (\forall V1n \in ty\_Enum\_Enum. ( \\ & \quad \forall V2p \in ty\_Enum\_Enum. ((ap\ (ap\ c\_Earithmetic\_E\_2B\ V0m) \\ & (ap\ (ap\ c\_Earithmetic\_E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_Earithmetic\_E\_2B \\ & \quad (ap\ (ap\ c\_Earithmetic\_E\_2B\ V0m)\ V1n))\ V2p)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum. ((V0m = c\_Enum\_E0) \vee (\exists V1n \in \\ & \quad ty\_Enum\_Enum. (V0m = (ap\ c\_Enum\_ESUC\ V1n)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum. (\forall V1n \in ty\_Enum\_Enum. ( \\ & \quad p\ (ap\ (ap\ c\_Earithmetic\_E\_3C\_3D\ V0m)\ (ap\ (ap\ c\_Earithmetic\_E\_2B \\ & \quad \quad V0m)\ V1n)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum. ((V1k = (ap ( \\
& ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A (ap (ap c\_2Earithmetic\_2EDIV \\
& V1k) V0n)) V0n)) (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n))) \wedge (p (ap \\
& (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n)) \\
& V0n))))))
\end{aligned} \tag{14}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Rightarrow (\neg (V0m = V1n))))))
\end{aligned} \tag{16}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2EDIV V1k) V0n)) V1k))))))
\end{aligned}$$