thm_2Earithmetic_2EDIV__MOD__MOD__DIV (TM-NoikeEyuPKsx5zWsmc3MBmQ7NppBJSjjY)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (3)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (7)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{9}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 14 *We define* c_2Emarker_2EAC *to be* $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ V0b1)$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V0m)))) \tag{10}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V2p))))) \tag{11}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0m)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V1n)))))) \tag{12}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow(\forall V1k \in ty\_2Enum\_2Enum.((V1k = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V1k)\ V0n))\ V0n))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n)))\land(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))\ V0n))))))\tag{13}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1k)\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V1k))))\tag{14}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow(\forall V1q \in ty\_2Enum\_2Enum.(\forall V2r \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1q)\ V0n))\ V2r))\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V2r)\ V0n))))))\tag{15}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow(\forall V1x \in ty\_2Enum\_2Enum.(\forall V2r \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1x)\ V0n))\ V2r))\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1x)\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V2r)\ V0n)))))))\tag{16}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2z \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2z))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V1y)\ V2z))\ V0x))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1y)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0x)\ V2z))))))))\tag{17}$$

Assume the following.

$$True\tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True\land(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\land True)\Leftrightarrow(p\ V0t))\land(((False\land(p\ V0t))\Leftrightarrow False)\land((((p\ V0t)\land False)\Leftrightarrow False)\land(((p\ V0t)\land(p\ V0t))\Leftrightarrow(p\ V0t)))))))\tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}). ((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a. ((ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z)) = (ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x)\ V2y))\ V3z))))) \Rightarrow ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((ap\ (ap\ V0f\ V4x)\ V5y) = (ap\ (ap\ V0f\ V5y)\ V4x)))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.(\forall V8z \in A\_27a.((ap\ (ap\ V0f\ V6x)\ (ap\ (ap\ V0f\ V7y)\ V8z)) = (ap\ (ap\ V0f\ V7y)\ (ap\ (ap\ V0f\ V6x)\ V8z))))))))) \tag{23}$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2k \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2k))) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0m)\ V1n))\ V2k) = (ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2k)))\ V1n))))))$$