thm_2Earithmetic_2EEVEN__EXP__IFF (TMcnXWhcnjMtYYNGiaW1Zg6sBywpqFM5fdF)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(7)

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(8)

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$
(9)

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 10 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 14 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 16 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V0m)\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V2n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1m)\ (ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V1m)\ V2n))))))$$
(10)

Assume the following.

$$(((p\ (ap\ c\_2Earithmetic\_2EEVEN\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow True) \wedge (\forall V0n \in$$
$$ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V0n))) \Leftrightarrow (\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n))))))$$
(11)

Assume the following.

$$((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})) = (ap\ \textsf{c\_2Enum\_2ESUC}\ \textsf{c\_2Enum\_2E0}))$$

(12)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)$$
$$V1n)))\Leftrightarrow((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0m))\vee(p\ (ap\ c\_2Earithmetic\_2EEVEN$$
$$V1n))))))$$

(13)

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p$$
$$V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2))))))$$

(15)

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t)))$$

(17)

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow$$
$$(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge$$
$$(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t)))))))$$

(18)

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge$$
$$(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee$$
$$(p\ V0t))\Leftrightarrow(p\ V0t)))))))$$

(19)

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge$$
$$((\neg False)\Leftrightarrow True)))$$

(20)

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow$$
$$(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg($$
$$p\ V0t)))))))$$

(21)

3

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p \ (ap \ V0P \ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p \ (ap \ V0P \ V1n)) \Rightarrow (p \ (ap \ V0P \ (ap \ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p \ (ap \ V0P \ V2n)))))$$

$$(22)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n) \ V0n))))$$

$$(23)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Enum\_2E0})$$
$$(ap \ \mathsf{c\_2Enum\_2ESUC} \ V0n))))$$

$$(24)$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$(p \ (ap \ c\_2Earithmetic\_2EEVEN \ (ap \ (ap \ c\_2Earithmetic\_2EEXP \ V1m)$$
$$V0n))) \Leftrightarrow ((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Enum\_2E0}) \ V0n)) \wedge (p$$
$$(ap \ c\_2Earithmetic\_2EEVEN \ V1m)))))))$$