

# thm\_2Earithmetic\_2EEVEN\_\_ODD\_\_EXISTS (TMWLjEr2Qyche5k5gUZfjrYXLJPzt9AdGoz)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ a))$

Let `c_2Enum_2EZERO__REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EZERO__REP \in \text{omega} \tag{1}$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{2}$$

Let `c_2Enum_2EABS__num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{3}$$

**Definition 4** We define `c_2Enum_2E0` to be  $(\text{ap } c_2Enum_2EABS__num \ c_2Enum_2EZERO__REP)$ .

Let `c_2Enum_2EREP__num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EREP__num \in (\text{omega}^{ty_2Enum_2Enum}) \tag{4}$$

Let `c_2Enum_2ESUC__REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2ESUC__REP \in (\text{omega}^{\text{omega}}) \tag{5}$$

**Definition 5** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ (\lambda V1x \in 2. V1x)) \ (\lambda V2x \in 2. V2x))$

**Definition 7** We define `c_2Enum_2ESUC` to be  $\lambda V0m \in ty_2Enum_2Enum. (\text{ap } c_2Enum_2EABS__num \ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0n) c\_2Enum\_2E0)$ .

**Definition 9** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B V0n) c\_2Enum\_2E0)$ .

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 12** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$ .

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$ .

Assume the following.

$$(((p (ap c\_2Earithmetic\_2EEVEN c\_2Enum\_2E0)) \Leftrightarrow True) \wedge (\forall V0n \in ty\_2Enum\_2Enum.((p (ap c\_2Earithmetic\_2EEVEN (ap c\_2Enum\_2ESUC V0n))) \Leftrightarrow (\neg (p (ap c\_2Earithmetic\_2EEVEN V0n))))))) \quad (10)$$

Assume the following.

$$((ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) = (ap c\_2Enum\_2ESUC c\_2Enum\_2E0)) \quad (11)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EODD V0n)) \Leftrightarrow \\
& (\neg(p (ap c\_2Earithmetic\_2EVEN V0n))))
\end{aligned} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p (ap V0P c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p (ap V0P V2n))))))
\end{aligned} \tag{20}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (((p (ap c\_2Earithmetic\_2EEVEN \\ & V0n)) \Rightarrow (\exists V1m \in ty\_2Enum\_2Enum. (V0n = (ap (ap c\_2Earithmetic\_2E\_2A \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\ & V1m)))) \wedge ((p (ap c\_2Earithmetic\_2EODD V0n)) \Rightarrow (\exists V2m \in ty\_2Enum\_2Enum. \\ & (V0n = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V2m))))))) \end{aligned}$$