thm_2Earithmetic_2EEXP__ALWAYS__BIG__ENOUGH (TMTa3JnuKuNu9XwaDkZT2PA3UhukG2KmECL)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\text{-}27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\text{-}27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\text{-}2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$ (

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 15 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$\tag{8}$$

Definition 16 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 17 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V0m)\ \textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$\textsf{c\_2Earithmetic\_2EZERO})))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1m)\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V2n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1m)\ (ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$V1m)\ V2n))))))$$
$$\tag{9}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = \textsf{c\_2Enum\_2E0}) \vee (\exists V1n \in$$
$$ty\_2Enum\_2Enum.(V0m = (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n))))) \tag{10}$$

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Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((\neg(V0n = \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)))) \tag{11}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n)))))$$
$$\tag{12}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{13}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \neg((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m) = \mathsf{c\_2Enum\_2E0})))) \tag{14}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m)) \wedge (\\ p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V1n))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))$$
$$\tag{15}$$

Assume the following.
$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((p \; V0t) \Rightarrow False) \Rightarrow (\neg(p \; V0t)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \; V0t)) \Rightarrow ((p \; V0t) \Rightarrow False))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \wedge True) \Leftrightarrow (p \; V0t)) \wedge (((False \wedge (p \; V0t)) \Leftrightarrow False) \wedge ((((p \; V0t) \wedge False) \Leftrightarrow False) \wedge (((p \; V0t) \wedge (p \; V0t)) \Leftrightarrow (p \; V0t))))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \; V0t)) \Leftrightarrow True) \wedge ((((p \; V0t) \Rightarrow (p \; V0t)) \Leftrightarrow True) \wedge (((p \; V0t) \Rightarrow False) \Leftrightarrow (\neg(p \; V0t)))))))))) \tag{24}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \; V0t))) \Leftrightarrow (p \; V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty \; A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty \; A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \Leftrightarrow True) \Leftrightarrow (p \; V0t)) \wedge (((False \Leftrightarrow (p \; V0t)) \Leftrightarrow (\neg(p \; V0t))) \wedge (((p \; V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \; V0t)))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty \; A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p \; (ap \; V0P \; V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p \; (ap \; V0P \; V2x)))))) \tag{29}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p \; (ap \; V0P \; \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p \; (ap \; V0P \; V1n)) \Rightarrow (p \; (ap \; V0P \; (ap \; \mathsf{c\_2Enum\_2ESUC} \; V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p \; (ap \; V0P \; V2n))))) \tag{30}$$

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Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n) \ \mathsf{c\_2Enum\_2E0})))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t)))\Leftrightarrow(p \ V0t))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A)\Rightarrow((\neg(p \ V0A))\Rightarrow False))) \tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A)\lor(p \ V1B)))\Rightarrow False)\Leftrightarrow$$
$$(((p \ V0A)\Rightarrow False)\Rightarrow((\neg(p \ V1B))\Rightarrow False))))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A))\lor(p \ V1B)))\Rightarrow False)\Leftrightarrow$$
$$((p \ V0A)\Rightarrow((\neg(p \ V1B))\Rightarrow False))))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A))\Rightarrow False)\Rightarrow(((p \ V0A)\Rightarrow False)\Rightarrow False))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p)\Leftrightarrow($$
$$(p \ V1q)\Leftrightarrow(p \ V2r)))\Leftrightarrow(((p \ V0p)\lor((p \ V1q)\lor(p \ V2r)))\land(((p \ V0p)\lor((\neg($$
$$p \ V2r))\lor(\neg(p \ V1q))))\land(((p \ V1q)\lor((\neg(p \ V2r))\lor(\neg(p \ V0p))))\land((p \ V2r)\lor$$
$$((\neg(p \ V1q))\lor(\neg(p \ V0p))))))))))))) \tag{37}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p)\Leftrightarrow($$
$$(p \ V1q)\Rightarrow(p \ V2r)))\Leftrightarrow(((p \ V0p)\lor(p \ V1q))\land(((p \ V0p)\lor(\neg(p \ V2r)))\land(($$
$$\neg(p \ V1q))\lor((p \ V2r)\lor(\neg(p \ V0p)))))))))))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p)\Leftrightarrow(\neg(p \ V1q)))\Leftrightarrow(((p \ V0p)\lor$$
$$(p \ V1q))\land((\neg(p \ V1q))\lor(\neg(p \ V0p)))))))) \tag{39}$$

Theorem 1

$$(\forall V0b \in ty\_2Enum\_2Enum.((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO})))$$
$$V0b))\Rightarrow(\forall V1n \in ty\_2Enum\_2Enum.(\exists V2m \in ty\_2Enum\_2Enum.$$
$$(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ V1n) \ (ap \ (ap \ c\_2Earithmetic\_2EEXP$$
$$V0b) \ V2m)))))))$$

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