

thm_2Earithmetic_2EEXP__BASE__MULT
(TMZcPXaW4Tc9PS5RhvsStVMRTPyMP3nGems)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{7}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E_21 2) V2t))))$

Definition 11 We define $c_2Emarker_2EAC$ to be $\lambda V0b1 \in 2. \lambda V1b2 \in 2. (ap (ap c_2Ebool_2E_2F_5C V0b1) V1b2)$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP \\ V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\ V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\ & \quad V1m) V2n)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & \quad (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & \quad V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & \quad (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & \quad \quad V0m) V1n))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\ & \quad V1n) V0m)))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A \\
& \quad (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p))))))
\end{aligned} \tag{12}$$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}). \\
& \quad ((\forall V1x \in A_27a. (\forall V2y \in A_27a. (\forall V3z \in A_27a. \\
& ((ap (ap V0f V1x) (ap (ap V0f V2y) V3z))) = (ap (ap V0f (ap (ap V0f V1x) \\
& \quad V2y)) V3z)))))) \Rightarrow ((\forall V4x \in A_27a. (\forall V5y \in A_27a. ((ap \\
& (ap V0f V4x) V5y) = (ap (ap V0f V5y) V4x)))) \Rightarrow (\forall V6x \in A_27a. (\\
& \quad \forall V7y \in A_27a. (\forall V8z \in A_27a. ((ap (ap V0f V6x) (ap (ap \\
& V0f V7y) V8z))) = (ap (ap V0f V7y) (ap (ap V0f V6x) V8z))))))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap V0P V2n))))))
\end{aligned} \tag{17}$$

Theorem 1

$$\begin{aligned}
& (\forall V0z \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \quad \forall V2y \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP (ap \\
& (ap c_2Earithmetic_2E_2A V1x) V2y)) V0z) = (ap (ap c_2Earithmetic_2E_2A \\
& (ap (ap c_2Earithmetic_2EEXP V1x) V0z)) (ap (ap c_2Earithmetic_2EEXP \\
& \quad V2y) V0z))))))
\end{aligned}$$