

thm_2Earithmetic_2EFUNPOW__ADD
(TMLGaqFrys48R4Q5vAc6FsyD51V9aGTdFRe)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in ((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})} \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{6}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{7}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0f \in (A_27a^{A_27a}).(\forall V1x \in \\ & A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_27a)\ V0f)\ c_2Enum_2E0) \\ & V1x) = V1x))) \wedge (\forall V2f \in (A_27a^{A_27a}).(\forall V3n \in ty_2Enum_2Enum. \\ & (\forall V4x \in A_27a.((ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_27a) \\ & V2f)\ (ap\ c_2Enum_2ESUC\ V3n))\ V4x) = (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW \\ & A_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (A_27a^{A_27a}).(\forall V1n \in \\ & ty_2Enum_2Enum.(\forall V2x \in A_27a.((ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW \\ & A_27a)\ V0f)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2x) = (ap\ V0f\ (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW \\ & A_27a)\ V0f)\ V1n)\ V2x))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (14)$$

Theorem 1

$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0f \in (A_{27a}^{A_{27a}}).(\forall V1x \in$
 $A_{27a}.(\forall V2m \in ty_2Enum_2Enum.(\forall V3n \in ty_2Enum_2Enum.$
 $((ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_{27a})\ V0f)\ (ap\ (ap\ c_2Earithmetic_2E_2B$
 $V2m)\ V3n))\ V1x) = (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_{27a})\ V0f)$
 $V2m)\ (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_{27a})\ V0f)\ V3n)\ V1x))))))$