

thm\_2Earithmetic\_2EINV\_\_PRE\_\_LESS  
 (TMboys4LswJ5kbAoaoCfSb8XEzrYR7ohukx)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x)) \text{ else } (\lambda x. x \in A \wedge \neg p x)$  of type  $\iota \Rightarrow \iota$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.\dots)))$

**Definition 10** We define  $c\_Ebool\_ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 11** We define  $c_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (ap (c_2Ebool_2B$

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E))$

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 14** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Assume the following.

$(\forall V0m \in$

$$(p \ (ap \ (ap \ c_2Eprim\_rec\_2E\_3C \ (ap \ c_2Enum\_2ESUC \ V0m)) \ (ap \ c_2Enum\_2ESUC \ V1n))) \Leftrightarrow (p \ (ap \ (ap \ c_2Eprim\_rec\_2E\_3C \ V0m) \ V1n)))) \quad (6)$$

Assume the following.

*True* (7)

Assume the following.

$$((\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \vee 0t1) \Rightarrow (p \vee 1t2)) \Rightarrow (((p \vee 1t2) \Rightarrow (p \vee 0t1)) \Rightarrow ((p \vee 0t1) \Leftrightarrow (p \vee 1t2))))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\;V0t))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg\text{True}) \Leftrightarrow \text{False}) \wedge ((\neg\text{False}) \Leftrightarrow \text{True}))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t))))))) \quad (11)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\ (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\ V1n))))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \quad (12)$$

Assume the following.

$$(((ap\ c\_2Eprim\_rec\_2EPRE\ c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (\forall V0m \in ty\_2Enum\_2Enum. ((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Enum\_2ESUC\ V0m)) = V0m))) \quad (13)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V0n)))) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ c\_2Enum\_2E0)))) \quad (15)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ (ap\ c\_2Enum\_2ESUC\ V0n)))) \quad (16)$$

### Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0m)) \Rightarrow (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Eprim\_rec\_2EPRE\ V0m))\ (ap\ c\_2Eprim\_rec\_2EPRE\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)))))))$$