

thm\_2Earithmetic\_2ELEFT\_\_ADD\_\_DISTRIB  
(TMSR2cBXSjHPqXPQ6rpZoJdBkMeRjTNTeU6)

October 26, 2020

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{6}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))\ (\lambda V3P \in 2.V3P))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2E\_2A V0n) c\_2Enum\_2E0) = c\_2Enum\_2E0$ .

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2A \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$ .

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B \\ & V0m) V1n)))) \wedge ((ap (ap c\_2Earithmic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & (ap (ap c\_2Earithmic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmic\_2E\_2B \\ & V1n) V0m)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2B V0m) \\ & (ap (ap c\_2Earithmic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmic\_2E\_2B \\ & (ap (ap c\_2Earithmic\_2E\_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmic\_2E\_2A (ap c\_2Earithmic\_2ENUMERAL \\ & (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)) V0m) = V0m) \wedge \\ & (((ap (ap c\_2Earithmic\_2E\_2A V0m) (ap c\_2Earithmic\_2ENUMERAL \\ & (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)) = V0m) \wedge ( \\ & ((ap (ap c\_2Earithmic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\ & (ap c\_2Earithmic\_2E\_2B (ap (ap c\_2Earithmic\_2E\_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c\_2Earithmic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\ & (ap (ap c\_2Earithmic\_2E\_2B V0m) (ap (ap c\_2Earithmic\_2E\_2A \\ & V0m) V1n)))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (14)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2p) \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2p)\ V0m))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ & V2p)\ V1n)))))) \end{aligned}$$