thm_2Earithmetic_2ELESS__EQ (TMRDZhnf2xxBzTArk5TwhgyRKBceFV8sD24)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 14 *We define* c_2Erelation_2ERTC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

Definition 15 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Assume the following.

$$(\text{c\_2Earithmetic\_2E\_3C\_3D} = (ap\ (\text{c\_2Erelation\_2ERTC}\ ty\_2Enum\_2Enum)\ (\lambda V0x \in ty\_2Enum\_2Enum.(\lambda V1y \in ty\_2Enum\_2Enum.(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V1y)\ (ap\ \text{c\_2Enum\_2ESUC}\ V0x)))))) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{A\_27a}).(\forall V1m \in A\_27a.(\forall V2n \in A\_27a.((p\ (ap\ (ap\ (ap\ (\text{c\_2Erelation\_2ERTC}\ A\_27a)\ (\lambda V3x \in A\_27a.(\lambda V4y \in A\_27a.(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ A\_27a)\ V4y)\ (ap\ V0f\ V3x)))))\ (ap\ V0f\ V1m))\ V2n)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (\text{c\_2Erelation\_2ETC}\ A\_27a)\ (\lambda V5x \in A\_27a.(\lambda V6y \in A\_27a.(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ A\_27a)\ V6y)\ (ap\ V0f\ V5x)))))\ V1m)\ V2n)))))) \quad (9)$$

Assume the following.

$$(\text{c\_2Eprim\_\_rec\_2E\_3C} = (ap\ (\text{c\_2Erelation\_2ETC}\ ty\_2Enum\_2Enum)\ (\lambda V0x \in ty\_2Enum\_2Enum.(\lambda V1y \in ty\_2Enum\_2Enum.(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V1y)\ (ap\ \text{c\_2Enum\_2ESUC}\ V0x)))))) \quad (10)$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ V1n)))))$$