

thm_2Earithmetic_2ELESS__EQ__IMP__LESS__SUC (TMb4b3bcrwMFrM2ERdpPuhKkNRp3jJYQX7u)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 11 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}.$

Definition 12 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in$

Definition 13 We define `c_2Earithmetic_2E_3C_3D` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}.$

Assume the following.

$$\begin{aligned} & (\forall V0n \in \text{ty_2Enum_2Enum}. (\forall V1m \in \text{ty_2Enum_2Enum}. (\\ & (p \text{ (ap (ap c_2Earithmetic_2E_3C_3D } V0n) \ V1m)) \Leftrightarrow (p \text{ (ap (ap c_2Eprim_rec_2E_3C} \\ & \quad V0n) \text{ (ap c_2Enum_2ESUC } V1m)))))) \end{aligned} \tag{5}$$

Assume the following.

$$\text{True} \tag{6}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p \ V0t)) \Leftrightarrow (p \ V0t))) \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \\ & \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge ((\\ & \quad (p \ V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg (p \ V0t)))))) \end{aligned} \tag{8}$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in \text{ty_2Enum_2Enum}. (\forall V1m \in \text{ty_2Enum_2Enum}. (\\ & (p \text{ (ap (ap c_2Earithmetic_2E_3C_3D } V0n) \ V1m)) \Rightarrow (p \text{ (ap (ap c_2Eprim_rec_2E_3C} \\ & \quad V0n) \text{ (ap c_2Enum_2ESUC } V1m)))))) \end{aligned}$$