

thm_2Earithmetic_2ELESS__EQ__LESS__EQ__MONO (TMTY4eVk1P3Lhd7fUhfJ8kVZUrcTuzidnsc)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V1n)\ V0m)))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V2p))\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V1n)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V1n)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V2p))) \Rightarrow (p\ (\\ & \quad ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V2p)))))) \end{aligned} \tag{8}$$

Theorem 1

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.(\forall V3q \in ty_2Enum_2Enum.((\\ & (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V2p)) \wedge (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & \quad V1n)\ V3q))) \Rightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V0m)\ V1n))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V2p)\ V3q)))))) \end{aligned}$$