

thm_2Earithmetic_2ELESS_EQ_MONO (TMHF7WJvoSCNaqD6Lqav8z7zkuTE9oGSa7e)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota.$

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 12 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}.$

Definition 13 We define `c_2Earithmetic_2E_3C_3D` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in \text{ty_2Enum_2Enum}. (\forall V1n \in \text{ty_2Enum_2Enum}. (\\ & (p \text{ (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC } V0m)) \text{ (ap c_2Enum_2ESUC } \\ & V1n)))) \Leftrightarrow (p \text{ (ap (ap c_2Eprim_rec_2E_3C } V0m) V1n)))) \end{aligned} \quad (5)$$

Assume the following.

$$\text{True} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A. 27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A. 27a. \text{nonempty } A. 27a \Rightarrow (\forall V0x \in A. 27a. ((V0x = V0x) \Leftrightarrow \\ & \text{True})) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in \text{ty_2Enum_2Enum}. (\forall V1n \in \text{ty_2Enum_2Enum}. (\\ & ((\text{ap c_2Enum_2ESUC } V0m) = (\text{ap c_2Enum_2ESUC } V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in \text{ty_2Enum_2Enum}. (\forall V1m \in \text{ty_2Enum_2Enum}. (\\ & (p \text{ (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC } V0n)) \text{ (ap } \\ & \text{c_2Enum_2ESUC } V1m)))) \Leftrightarrow (p \text{ (ap (ap c_2Earithmetic_2E_3C_3D } V0n) \\ & V1m)))) \end{aligned}$$