

thm_2Earithmetic_2ELESS__IMP__LESS__ADD (TMTa7cVpk74u6FQTnQkLzk9zkHfB36rQgi1)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge ((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \forall V2p \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1n)\ V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\ & (V0m = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V2p)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) (ap c_2Enum_2ESUC V1n)))))) \quad (11)
 \end{aligned}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m)) \Rightarrow (\forall V2p \in ty_2Enum_2Enum. \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap (ap c_2Earithmetic_2E_2B \\
 & V1m) V2p))))))
 \end{aligned}$$