thm_2Earithmetic_2ELESS_MONO_ADD (TMXxTpwg6iQBuAwaRifzpg2B1YrLk3Z3gSv)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \; ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \; (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap \; (ap \; (\mathsf{c\_2Emin\_2E\_3D} \; (2^2)) \; (\lambda V0x \in 2.V0x)) \; (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \; (ap \; (\mathsf{c\_2Emin\_2E\_3D} \; (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap \; (\mathsf{c\_2Ebool\_2E\_21} \; 2) \; (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o \; (p \; P \Rightarrow p \; Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap \; (ap \; \mathsf{c\_2Emin\_2E\_3D\_3D\_3E} \; V0t) \; \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \; (\mathsf{c\_2Ebool\_2E\_21} \; 2) \; (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

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Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \land p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Definition 12 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \textsf{c\_2Enum\_2E0})\ V0m) = V0m) \land (((ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \textsf{c\_2Enum\_2E0}) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
(ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0m)\ V1n))) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\\
V1n)) = (ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))))\\
\tag{7}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ (ap\ \textsf{c\_2Enum\_2ESUC}\\
V1n))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))))\\
\tag{8}$$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow\\
(p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
p\ V0t)))))))) \tag{10}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \textsf{c\_2Enum\_2E0})) \land\\
(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \textsf{c\_2Enum\_2ESUC}\\
V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))\\
\tag{11}$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\\
V1n)) \Rightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\
V0m)\ V2p))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)))))))))$$