

thm\_2Earithmetic\_2ELESS\_\_SUB\_\_ADD\_\_LESS  
 (TMVb-  
 voYJS9QDAnWfcJ8VyGbwQ6vQ8aGL4Vv)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega^{\omega^a}}) \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega^{\omega^a}}) \quad (6)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0)) \wedge (\forall V1m \in ty\_2Enum\_2Enum. \\ & (\forall V2n \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ ( \\ & ap\ c\_2Enum\_2ESUC\ V1m))\ V2n) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Enum\_2Enum) \\ & (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V2n))\ c\_2Enum\_2E0)\ (ap\ c\_2Enum\_2ESUC \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ V2n)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (\neg (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ & V1n)\ V0m)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & V0m)\ c\_2Enum\_2E0) = V0m))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Earithmic\_2E\_3C\_3D V1n) V0m)) \Rightarrow ((ap (ap c\_2Earithmic\_2E\_2B (ap (ap c\_2Earithmic\_2E\_2D V0m) V1n)) V1n) = V0m)))))) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ c\_2Enum\_2E0)))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ (ap\ c\_2Enum\_2ESUC\ V1n)))) \Leftrightarrow ( \\ & (V0m = V1n) \vee (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)))))) \end{aligned} \quad (24)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\ & \forall V2i \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i) \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m))) \Rightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2i)\ V1m))\ V0n)))))) \end{aligned}$$