thm_2Earithmetic_2ELE_SUB_LCANCEL (TM-MeqsYSeC64gsSWXqj4iEuyG4TjAJcWJmm)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (4)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ (p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \text{c\_2Enum\_2ESUC}\ V0n))\ (ap\\ \text{c\_2Enum\_2ESUC}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\\ V1m))))) \tag{7}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\\ \text{c\_2Enum\_2E0})\ V0n))) \tag{8}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\\ (ap\ \text{c\_2Enum\_2ESUC}\ V0n))\ \text{c\_2Enum\_2E0})))) \tag{9}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\ \text{c\_2Enum\_2E0})\ V0m) = \text{c\_2Enum\_2E0})\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\ V0m)\ \text{c\_2Enum\_2E0}) = V0m))) \tag{10}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \text{c\_2Enum\_2ESUC}\ V0n))\ (ap\ \text{c\_2Enum\_2ESUC}\\ V1m)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m)))) \tag{11}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\\ V0m)\ V0m))) \tag{12}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (p \ (ap \ (ap \ \textsf{c\_2Eprim\_\_rec\_2E\_3C} \ V0m) \ V1n)) \Rightarrow (p \ (ap \ (ap \ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V0m) \ V1n)))))) \tag{13}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ p \ (ap \ (ap \ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \\ V0n) \ V1m)) \ V0n)))) \tag{14}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ (\neg(p \ (ap \ (ap \ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \textsf{c\_2Enum\_2ESUC} \ V0n)) \ V1m))) \Leftrightarrow \\ (p \ (ap \ (ap \ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \ V1m) \ V0n)))))) \tag{15}$$

Assume the following.

$$(\forall V0p \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ p \ (ap \ (ap \ \textsf{c\_2Eprim\_\_rec\_2E\_3C} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V0p) \\ V1m)) \ (ap \ \textsf{c\_2Enum\_2ESUC} \ V0p)))))) \tag{16}$$

Assume the following.
$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p \ V0t)) \Leftrightarrow True) \land ((((p \ V0t) \lor True) \Leftrightarrow True) \land \\ (((False \lor (p \ V0t)) \Leftrightarrow (p \ V0t)) \land ((((p \ V0t) \lor False) \Leftrightarrow (p \ V0t)) \land (((p \ V0t) \lor \\ (p \ V0t)) \Leftrightarrow (p \ V0t))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \land ((((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \land (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \land (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ p \ V0t))))))) \tag{22}$$

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Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \textsf{c\_2Enum\_2E0}))\wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \textsf{c\_2Enum\_2ESUC}$$
$$V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))) \tag{23}$$

Theorem 1

$$(\forall V0z \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.($$
$$\forall V2x \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2x)\ V1y))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V2x)\ V0z)))\Leftrightarrow((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0z)\ V1y))\vee(p$$
$$(ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2x)\ V1y)))))))$$