

thm_2Earithmetic_2ELT__MULT__CANCEL__RBARE (TMFRHGh321n9gM5UrzXU32dp9fcXTLjTZfC)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ P))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow p \ Q)$ of type $\iota.$

Definition 5 We define `c_2Ebool_2E_T` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a} \ P))$

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2. V2t))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{1}$$

Let `c_2Earithmetic_2E_2B` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let `c_2Enum_2EZERO_REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\text{omega}} \tag{4}$$

Definition 8 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP).$

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_2D_2D_2E\ V0t)\ c_2Ebool_2E$

Definition 16 We define $c_2Eprim_rec_2E_23C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \quad (8)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (9)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_23C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_23C\ V0m)\ V1n)))))) \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (\\
& ap (ap c_2Earithmetic_2E_2A V0m) V1n)) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V2p))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge \\
& (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (\\
& ap (ap c_2Earithmetic_2E_2A V0m) V1n)) (ap (ap c_2Earithmetic_2E_2A \\
& V2p) V1n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n)) \wedge \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0m) V2p))))))
\end{aligned} \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{16}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
((\neg False) \Leftrightarrow True))) \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ c_2Enum_2E0)))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ c_2Enum_2ESUC\ V0n)))) \quad (22)$$

Theorem 1

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V0m)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V0m)) \wedge (V1n = c_2Enum_2E0))) \wedge ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V1n)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V1n)) \wedge (V0m = c_2Enum_2E0))))))$$