

thm_2Earithmetic_2EMIN_0 (TMQKP-pHoT3kVZyABAf7HKo5nxYJQJKJDcK8)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (3)$$

Let $c_2Enum_2EAABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EAABS_num m)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^A \rightarrow 27a).(\lambda P V0P \ P(c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earthmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 14 We define $c_{_2Ebool_2ECOND}$ to be $\lambda A._27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A._27a.(\lambda V2t2 \in A._27a.($

Definition 15 We define $c_2Earthmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2E\text{Enum}_2EZERO_REP : \iota$ be given. Assume the following.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V1n) V0m))))) \quad (6)$$

Assume the following.

True (7)

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.((p\;V0t) \vee (\neg(p\;V0t)))) \quad (10)$$

Assume the following.

$$((\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.((p V0t1) \wedge (p V1t2) \wedge (p V2t3)))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((p\;V0t) \Rightarrow False) \Rightarrow (\neg(p\;V0t)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (16)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (20)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))))) \quad (21)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x)))))))) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & (\forall V0P \in (2^{A_{_27a}}).(\forall V1Q \in \\ (2^{A_{_27a}}).((\exists V2x \in A_{_27a}.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_{_27a}.(p (ap V0P V3x))) \vee (\exists V4x \in A_{_27a}.(p (ap V1Q V4x))))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))) \wedge \\ ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (p V1B)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & \forall A_{_27b}.nonempty\ A_{_27b} \Rightarrow \\ \forall V0b \in 2.(\forall V1f \in (A_{_27b}^{A_{_27a}}).(\forall V2g \in (A_{_27b}^{A_{_27a}}). \\ (\forall V3x \in A_{_27a}.((ap (ap (ap (c_2Ebool_2ECOND (A_{_27b}^{A_{_27a}}) \\ V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_{_27b}) V0b) (ap \\ V1f V3x)) (ap V2g V3x))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & \forall A_{_27b}.nonempty\ A_{_27b} \Rightarrow \\ \forall V0f \in (A_{_27b}^{A_{_27a}}).(\forall V1b \in 2.(\forall V2x \in A_{_27a}. \\ (\forall V3y \in A_{_27a}.((ap V0f (ap (ap (c_2Ebool_2ECOND A_{_27a}) \\ V1b) V2x) V3y) = (ap (ap (ap (c_2Ebool_2ECOND A_{_27b}) V1b) (ap V0f \\ V2x)) (ap V0f V3y))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.((p (ap (ap \\ (ap (c_2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge \\ ((p V0b) \vee (p V2t2))))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C \\ V0n) c_2Enum_2E0)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee ((\neg(p V1q)) \vee ((\neg(p V0p)) \vee ((\neg(p V1q) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (40)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EMIN V0n) c_2Enum_2E0) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2EMIN c_2Enum_2E0) V0n) = c_2Enum_2E0)))$$