

thm_2Earithmetic_2EMODEQ_0 (TMMjhSUBa-jYg3rwGF6FaE5rQdjAKgSwaTHc)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be ($ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP$).

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be ($ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$)

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) V0)$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EA \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 9 We define $c_{\text{2Emin_40}}$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \text{ then } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota)$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(\text{ap } V0P \text{ (ap (c_2Emin_2E_40}$

Definition 11 We define $c_2Earithmetic_2EMODEQ$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1m1 \in ty_2Enum.$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\ t \in 2.V0t))$.

Definition 13 We define $c_2 \in \text{min}_2 \in \text{D}_3 \text{D}_3 \text{E}$ to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E))$

Definition 15 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) V0n) = c_2Enum_2E0))) \quad (10)$$

Assume the following.

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$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m1 \in ty_2Enum_2Enum. \\
 & (\forall V2m2 \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & c_2Enum_2E0) V0n)) \Rightarrow ((p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) \\
 & V1m1) V2m2)) \Leftrightarrow ((ap (ap c_2Earithmetic_2EMOD V1m1) V0n) = (ap (ap \\
 & c_2Earithmetic_2EMOD V2m2) V0n))))))) \\
 \end{aligned} \tag{12}$$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
 & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))))
 \end{aligned} \tag{14}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\
 & (p V0t)))))))
 \end{aligned} \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow ((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \tag{18}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
 & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\
 & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))
 \end{aligned} \tag{19}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & c_2Enum_2E0) V0n)) \Rightarrow (p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) \\
 & V0n) c_2Enum_2E0))))) \\
 \end{aligned}$$