

thm_2Earithmetic_2EMODEQ__EXP__CONG
(TMGx7GdtFq3yHaZ3y345DSMsGe3PUYLvbCP)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Earithmetic_2EMODEQ$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1m1 \in ty_2Enum$

Definition 12 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\ & V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\ & V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\ & V1m) V2n)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1x0 \in ty_2Enum_2Enum. \\ & (\forall V2x1 \in ty_2Enum_2Enum.(\forall V3y0 \in ty_2Enum_2Enum. \\ & (\forall V4y1 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2Earithmetic_2EMODEQ \\ & V0n) V1x0) V2x1)) \Rightarrow ((p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) V3y0) \\ & V4y1)) \Rightarrow (p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) (ap (ap c_2Earithmetic_2E_2A \\ & V1x0) V3y0)) (ap (ap c_2Earithmetic_2E_2A V2x1) V4y1)))))))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Enum_2Enum.(p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) V1x) V1x)))) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty.2Enum.2Enum}). (((p (ap V0P c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty.2Enum.2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c.2Enum.2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty.2Enum.2Enum. (p (ap V0P V2n)))))) \end{aligned} \quad (18)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty.2Enum.2Enum. (\forall V1x \in ty.2Enum.2Enum. (\\ & \forall V2y \in ty.2Enum.2Enum. (\forall V3e \in ty.2Enum.2Enum. (\\ & p (ap (ap (ap c.2Earithmetic.2EMODEQ V0n) V1x) V2y)) \Rightarrow (p (ap (ap (\\ & ap c.2Earithmetic.2EMODEQ V0n) (ap (ap c.2Earithmetic.2EEXP V1x) \\ & V3e)) (ap (ap c.2Earithmetic.2EEXP V2y) V3e)))))) \end{aligned}$$