

# thm\_2Earithmetic\_2EMODEQ\_\_MOD (TMX- uxUA7CREedrmwojyAq3Nx9fRM3FacwtJ)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p (ap\ P\ x))$  then  $(the\ (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}})).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A_{27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}})).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A_{27a}})))$

**Definition 6** We define  $c\_2Earithmetic\_2EMODEQ$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1m1 \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

**Definition 11** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{omega}) \quad (8)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2ESUC\_REP\ V0m)$

**Definition 13** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n)))))) \\ & \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m1 \in ty\_2Enum\_2Enum. \\ & (\forall V2m2 \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow ((p\ (ap\ (ap\ (ap\ c\_2Earithmetic\_2EMODEQ\ V0n)\ V1m1)\ V2m2)) \Leftrightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1m1)\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V2m2)\ V0n))))))) \\ & \quad (10) \end{aligned}$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \\ & \quad (12) \end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (17)$$

**Theorem 1**

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow (p\ (ap\ (ap\ (ap\ c\_2Earithmetic\_2EMODEQ\ V0n)\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1x)\ V0n))\ V1x))))$$