

thm_2Earithmic_2EMODEQ__NUMERAL
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UemE8JiMQbao4MuPA8ww34s5jEefNUk)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (ap c_2Enum_2EREP_num (ap c_2Enum_2ESUC_REP m)))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 17 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{7}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{8}$$

Definition 18 We define $c_2Earithmetic_2EMODEQ$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1m1 \in ty_2Enum$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD \\
& \quad (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) V0n) = (ap (ap c_2Earithmetic_2EMOD \\
& \quad \quad V1k) V0n))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m1 \in ty_2Enum_2Enum. \\
& \quad (\forall V2m2 \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad \quad c_2Enum_2E0) V0n)) \Rightarrow ((p (ap (ap (ap c_2Earithmetic_2EMODEQ V0n) \\
& \quad V1m1) V2m2)) \Leftrightarrow ((ap (ap c_2Earithmetic_2EMOD V1m1) V0n) = (ap (ap \\
& \quad \quad c_2Earithmetic_2EMOD V2m2) V0n)))))))
\end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& \quad \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& \quad \quad p V0t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& \quad ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{18}$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \Rightarrow (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Enum_2ESUC V0n)))) \quad (20)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V0n)) (ap c_2Earithmic_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c_2Earithmic_2EMODEQ (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V1m)) (ap (ap c_2Earithmic_2EMOD (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V1m)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V0n)))))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V0n)) (ap c_2Earithmic_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c_2Earithmic_2EMODEQ (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V1m)) (ap (ap c_2Earithmic_2EMOD (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V1m)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V0n)))))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V0n)) (ap c_2Earithmic_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c_2Earithmic_2EMODEQ (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V1m)) (ap (ap c_2Earithmic_2EMOD (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V1m)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V0n)))))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL V0n)) (ap c_2Earithmic_2ENUMERAL V1m))) \Rightarrow (p (ap (ap (ap c_2Earithmic_2EMODEQ (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V1m)) (ap (ap c_2Earithmic_2EMOD (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V1m)) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V0n)))))))))$$