

thm\_2Earithmetic\_2EMOD\_2 (TMW-  
pLGEmWKXVzmX3ycqyAUTsxRyGSTMDXzF)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ P))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 6** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 n) V0)$

Let  $c_2Earithmetic_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 7** We define  $c\_2EArithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 8** We define  $c\_2Earthmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earthmetic\_2EBIT2$

**Definition 9** We define  $c_2\text{Earithmetic\_ENUMERAL}$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Let  $c_2Earithmetic_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c_2Earithmetic_2E_2A : \iota$  be given. Assume the following.

$$c_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 10** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c_{\text{2Emin\_3D\_3D\_3E}}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 12** We define  $c_{\text{Ebool\_2E\_2F\_5C}}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{Ebool\_2E\_21}})2))(\lambda V2t \in$

**Definition 13** We define  $c_{\text{2Emin\_2E\_40}}$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then} \ (\lambda x. x \in A \wedge$   
 $\text{of type } \iota \Rightarrow \iota.$

**Definition 14** We define  $c\_Ebool\_ECOND$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 15** We define  $c_{\_2Ebool\_2E\_3F}$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^A \rightarrow 27a)).(ap\ V0P\ (ap\ (c_{\_2Emin\_2E\_40}$

**Definition 16** We define  $c_{\text{Ebool\_2E\_7E}}$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_{\text{Emin\_2E\_3D\_3D\_3E}}\ V0t)\ c_{\text{Ebool\_2E}}))$

**Definition 17** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 18** We define  $c_{\text{CBool}} : \mathbb{C}_5 \rightarrow \mathbb{F}$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_{\text{CBool}}_2)_{\text{C}_2} 2) (\lambda V2t \in$

Assume the following.

$$((ap\ c\_2Earthmetic\_2ENUMERAL\ (ap\ c\_2Earthmetic\_2EBIT1\ c\_2Earthmetic\_2EZERO)) = \\ (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0)) \quad (11)$$

Assume the following.

$$\begin{aligned} ((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)) = \\ (ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ c\_2Enum\_2E0) = V0m)) \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Enum\_2ESUC \\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ V1n)\ V0m)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n)) \Leftrightarrow \\ (\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ V0n)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EODD\ V0n)) \Leftrightarrow \\ (\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n)) \Leftrightarrow \\ (\exists V1m \in ty\_2Enum\_2Enum.(V0n = (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)) \\ V1m)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EODD V0n)) \Leftrightarrow \\
 & (\exists V1m \in ty\_2Enum\_2Enum. (V0n = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2A \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
 & V1m)))))) \\
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
 & \forall V2r \in ty\_2Enum\_2Enum. ((\exists V3q \in ty\_2Enum\_2Enum. ( \\
 & (V1k = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V3q) V0n)) V2r)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2r) V0n)))) \Rightarrow ( \\
 & (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n) = V2r)))))) \\
 \end{aligned} \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
 & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
 & (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (31)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\forall V2x \in A\_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A\_27a.((p (ap V0P V3x)) \vee (\exists V4x \in A\_27a.(p (ap V1Q V4x)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A\_27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ A\_27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).(((p V0P) \vee (\exists V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ A\_27a.((p V0P) \vee (p (ap V1Q V3x))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in ( \\ 2.((\exists V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ A\_27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).((\exists V2x \in A\_27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A\_27a.(p (ap V1Q V3x))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).((\forall V2x \in A\_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A\_27a.(p (ap V1Q V3x))))))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ( \\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee ( \\ (p V0A)))))) \quad (45)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\ p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ \forall V0b \in 2.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2g \in (A\_27b^{A\_27a}). \\ (\forall V3x \in A\_27a.((ap (ap (ap (c\_2Ebool\_2ECOND (A\_27b^{A\_27a}) \\ V0b) V1f) V2g) V3x) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V0b) (ap \\ V1f V3x)) (ap V2g V3x))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \\
& \quad \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1b \in 2. (\forall V2x \in A_{27a}. \\
& \quad (\forall V3y \in A_{27a}. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27a}) \\
& \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_{2Ebool\_2ECOND}\ A_{27b})\ V1b)\ (ap\ V0f\ \\
& \quad V2x))\ (ap\ V0f\ V3y))))))) \\
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p\ (ap\ (ap\ \\
& \quad (ap\ (c_{2Ebool\_2ECOND}\ 2)\ V0b)\ V1t1)\ V2t2)) \Leftrightarrow (((\neg(p\ V0b)) \vee (p\ V1t1)) \wedge \\
& \quad ((p\ V0b) \vee (p\ V2t2))))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \\
& \quad \forall V0P \in ((2^{A_{27b}})^{A_{27a}}). ((\forall V1x \in A_{27a}. (\exists V2y \in \\
& \quad A_{27b}. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}). ( \\
& \quad \forall V4x \in A_{27a}. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \\
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c_{2Eprim\_rec\_2E\_3C}\ c_{2Enum\_2E0}) \\
(ap\ c_{2Enum\_2ESUC}\ V0n)))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{66}$$

**Theorem 1**

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EMOD V0n) \\(ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) = \\(ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap c\_2Earithmetic\_2EEVEN \\V0n)) c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\c\_2Earithmetic\_2EZERO))))))$$