thm_2Earithmetic_2EMOD__P (TMJbrtf9zrEGVuyd2zfAVMTkyrRFVWyhMg8)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)$ $(\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\text{-}2Enum\_2Enum}) \qquad (6)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (7)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\textsf{c\_2Enum\_2E0})\ V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum.((V1k = (ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2EDIV$$
$$V1k)\ V0n))\ V0n))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))) \wedge (p\ (ap$$
$$(ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))$$
$$V0n))))))$$
$$(10)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.($$
$$\forall V2r \in ty\_2Enum\_2Enum.((\exists V3q \in ty\_2Enum\_2Enum.($$
$$(V1k = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V3q)\ V0n))\ V2r)) \wedge (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2r)\ V0n)))) \Rightarrow ($$
$$(ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V2r)))))$$
$$(11)$$

Assume the following.
$$True \qquad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p$$
$$V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))$$
$$(13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{16}$$

Theorem 1

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1p \in ty\_2Enum\_2Enum.$$
$$(\forall V2q \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$\mathsf{c\_2Enum\_2E0})\ V2q)) \Rightarrow ((p\ (ap\ V0P\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1p)$$
$$V2q))) \Leftrightarrow (\exists V3k \in ty\_2Enum\_2Enum.(\exists V4r \in ty\_2Enum\_2Enum.$$
$$((V1p = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V3k)\ V2q))\ V4r)) \land ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V4r)\ V2q)) \land (p$$
$$(ap\ V0P\ V4r)))))))))))))$$