

# thm\_2Earithmetic\_2EMULT\_\_DIV

(TMT5sR41bS8z9HYxCKE6eSBf94jTSJcbo83)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V0t \in 2.V0t)))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c_2Emin_2E_3D_3D_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E_21 2))(\lambda V2t \in 2.$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega) \quad (8)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

**Definition 10** We define  $c_2Emin_2E_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p \text{ (ap } P \text{ } x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \rightarrow \iota).$

**Definition 11** We define  $c_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^A)^{27a}).(ap\ V0P\ (ap\ (c_2Emin\_2E\ 40$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\\ c\_2Enum\_2E0) = V0m)) \quad (9)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1r \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c_2Eprim_rec_E_3C V1r) V0n)) \Rightarrow (\forall V2q \in ty\_2Enum\_2Enum. \\
 & ((ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2E_2B (ap \\
 & (ap c_2Earithmetic_2E_2A V2q) V0n) V1r) V0n) = V2q)))))) \\
 \end{aligned} \tag{10}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))))) \quad (11)$$

### Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1q \in ty\_2Enum\_2Enum. (\\(p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \ V0n)) \Rightarrow ((ap \ (ap \ c\_2Earithmetic\_2EDIV \ (ap \ (ap \ c\_2Earithmetic\_2E\_2A \ V1q) \ V0n)) \ V0n) = V1q))))$$