thm_2Earithmetic_2ENRC__0 (TMb-jdXMVK1iW49TJffsVHmhm4T2qSuLcZxG)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (5)$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Earithmetic\_2ENRC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2ENRC\ A\_27a \in ((((2^{A\_27a})^{A\_27a})^{ty\_2Enum\_2Enum})^{((2^{A\_27a})^{A\_27a})}) \tag{6}$$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Earithmetic\_2ENRC\ A\_27a)\ V0R)\ \mathsf{c\_2Enum\_2E0})\ V1x)\ V2y)) \Leftrightarrow (V1x = V2y))))) \wedge (\forall V3R \in ((2^{A\_27a})^{A\_27a}).(\forall V4n \in ty\_2Enum\_2Enum.(\forall V5x \in A\_27a.(\forall V6y \in A\_27a.((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Earithmetic\_2ENRC\ A\_27a)\ V3R)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V4n))\ V5x)\ V6y)) \Leftrightarrow (\exists V7z \in A\_27a. ((p\ (ap\ (ap\ V3R\ V5x)\ V7z)) \wedge (p\ (ap\ (ap\ (ap\ (ap\ (c\_2Earithmetic\_2ENRC\ A\_27a)\ V3R)\ V4n)\ V7z)\ V6y)))))))))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Earithmetic\_2ENRC\ A\_27a)\ V0R)\ \mathsf{c\_2Enum\_2E0})\ V1x)\ V2y)) \Leftrightarrow (V1x = V2y)))))$$