

thm\_2Earithmetic\_2ENRC\_RTC  
 (TMF8cBCVJrLKbhfYFV6Xb6oRJZT9NGUgRfa)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A) P)))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2ENRC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2ENRC\ A\_27a \in ((((2^{A-27a})^{A-27a})^{ty\_2Enum\_2Enum})^{((2^{A-27a})^{A-27a})}) \quad (2)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (3)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (5)$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}) P)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (m))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (6)$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EAABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c_2Emin_2E_3D_3D_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Erelation\_2ERTC$  to be  $\lambda A.\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2$

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$ .

**Definition 12** We define  $c_2Eb0l_2E_5C_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Eb0l_2E_21 2)))(\lambda V2t \in$

**Definition 13** We define  $c_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c_2Emin\_2E_3D\_3D\_3E V0t) c_2Ebool\_2E))$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0R \in ((2^{A\_27a})^{A\_27a})) \\
& (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p (ap (ap (ap (ap (c\_2Earithmetic\_2ENRC \\
& A\_27a) V0R) c\_2Enum\_2E0) V1x) V2y)) \Leftrightarrow (V1x = V2y)))))) \wedge (\forall V3R \in \\
& ((2^{A\_27a})^{A\_27a}).(\forall V4n \in \text{ty\_2Enum\_2Enum}.(\forall V5x \in \\
& A\_27a.(\forall V6y \in A\_27a.((p (ap (ap (ap (ap (c\_2Earithmetic\_2ENRC \\
& A\_27a) V3R) (ap c\_2Enum\_2ESUC V4n)) V5x) V6y)) \Leftrightarrow (\exists V7z \in A\_27a. \\
& ((p (ap (ap V3R V5x) V7z)) \wedge (p (ap (ap (ap (ap (c\_2Earithmetic\_2ENRC \\
& A\_27a) V3R) V4n) V7z) V6y))))))))))) \\
& (7)
\end{aligned}$$

Assume the following.

*True* (8)

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))) \quad (9)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (10)$$

Assume the following.

$$\forall A \_27a. nonempty \_A \_27a \Rightarrow (\forall V0x \in A \_27a. (\forall V1y \in A \_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ (2^{A\_27a}).((\forall V2x \in A\_27a.(p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ ((\forall V3x \in A\_27a.(p (ap V0P V3x)) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow \\ (\forall V3x \in A\_27a.(p V0P) \wedge (p (ap V1Q V3x)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\ ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\ (\forall V1n \in ty\_2Enum\_2Enum.(p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ ((\forall V1x \in A\_27a.(p (ap (ap (ap (c\_2ERelation\_2ERTC A\_27a) \\ V0R) V1x) V1x))) \wedge (\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(\forall V4z \in \\ A\_27a.(((p (ap (ap V0R V2x) V3y)) \wedge (p (ap (ap (ap (c\_2ERelation\_2ERTC \\ A\_27a) V0R) V3y) V4z))) \Rightarrow (p (ap (ap (ap (c\_2ERelation\_2ERTC A\_27a) \\ V0R) V2x) V4z)))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (30)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2x \in A\_27a.(\forall V3y \in \\ & A\_27a.((p (ap (ap (ap (ap (c\_2Earithmetic\_2ENRC A\_27a) V0R) V1n) \\ & V2x) V3y)) \Rightarrow (p (ap (ap (ap (c\_2Erelation\_2ERTC A\_27a) V0R) V2x) V3y))))))) \end{aligned}$$