

thm_2Earithmetic_2EONE_LT_MULT
 (TMXzBSAsqsNGccQbwH-
 MagS5HwSwKGVdv5GN)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota)$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y) \text{ of type } \iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A) a)))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$.

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})) a)))$.

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q) \text{ of type } \iota$.

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$.

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E_7E V2t) c_2Ebool_2EF))))))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^\omega) \quad (4)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Definition 15 We define $c_2Earithmetic_2EZERO$ to be $c_2Enum_2E0.$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E$

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) = \\ & \quad (ap c_2Enum_2ESUC c_2Enum_2E0)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (\\ & \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (\\ & \quad ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & \quad V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \end{aligned} \quad (9)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n))))) \quad (10)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Enum_2ESUC V0m)) (ap c_2Enum_2ESUC V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) \quad (11)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0n)) c_2Enum_2E0)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n)))))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2E_2B V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \vee (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & \quad (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & \quad (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ (p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\ & \quad (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) V0n)))) \quad (28)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Enum_2ESUC V0n)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge (\neg(p V2r)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \wedge (\neg(p V1q)))) \wedge (((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (40)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\ & \quad (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (\\ & \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) (ap (ap c_2Earithmetic_2E_2A \\ & \quad V0x) V1y))) \Leftrightarrow (((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0x)) \wedge \\ & \quad (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (\\ & \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V1y))) \vee (\\ & \quad (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1y)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\ & \quad V0x))))))) \end{aligned}$$