

thm_2Earithmetic_2EONE_LT_MULT_IMP (TMZRzLmyxT98PbAB4ds9SrMieQTcgmZbinb)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \text{ (ap } P \ x))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Let `c_2Enum_2EREP_num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{\text{ty_2Enum_2Enum}}) \tag{2}$$

Let `c_2Enum_2ESUC_REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \tag{4}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a}) \ (\lambda V1x \in 2. V1x))$

Definition 6 We define `c_2Enum_2ESUC` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. (\text{ap } c_2Enum_2EABS_num \text{ (ap } P \ x))$

Let `c_2Enum_2EZERO_REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \text{omega} \tag{5}$$

Definition 7 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Let $c_Earithmetic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (6)$$

Definition 9 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic_E_2B V0n) c_Enum_E0)$.

Definition 10 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (7)$$

Definition 11 We define c_Ebool_EF to be $(ap (c_Ebool_E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_Emin_E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_21) 2)$.

Definition 14 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in 2.V2t)))$.

Definition 15 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.V1n$.

Definition 16 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in 2.V2t)))$.

Assume the following.

$$((ap c_Earithmetic_ENUMERAL (ap c_Earithmetic_EBIT1 c_Earithmetic_EZERO)) = (ap c_Enum_ESUC c_Enum_E0)) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Enum_Enum.(\forall V1n \in ty_Enum_Enum.(\\ & ((ap (ap c_Earithmetic_E_2B c_Enum_E0) V0m) = V0m) \wedge (((ap (\\ & ap c_Earithmetic_E_2B V0m) c_Enum_E0) = V0m) \wedge (((ap (ap c_Earithmetic_E_2B \\ & (ap c_Enum_ESUC V0m)) V1n) = (ap c_Enum_ESUC (ap (ap c_Earithmetic_E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_Earithmetic_E_2B V0m) (ap c_Enum_ESUC \\ & V1n)) = (ap c_Enum_ESUC (ap (ap c_Earithmetic_E_2B V0m) V1n)))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_Enum_Enum.(\forall V1n \in ty_Enum_Enum.(\\ & \forall V2p \in ty_Enum_Enum.((ap (ap c_Earithmetic_E_2B V0m) \\ & (ap (ap c_Earithmetic_E_2B V1n) V2p)) = (ap (ap c_Earithmetic_E_2B \\ & (ap (ap c_Earithmetic_E_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (11)$$

Assume the following.

$$(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\ & ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge \\ & (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0m) = V0m) \wedge \\ & (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap \\ & (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n)) \\ & V1n)) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & V0m)\ V1n)))))) \quad (13) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum. \\ & (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m)) \Rightarrow (\forall V2p \in ty_2Enum_2Enum. \\ & (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V1m)\ V2p)))))) \quad (14) \end{aligned}$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (16) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \quad (17) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (22)$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg(p (ap (ap c.2Eprim_rec.2E.3C V0n) V0n)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{37}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0p)) \wedge (\\
& p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1q))) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& (ap (ap c_2Earithmetic_2E_2A V0p) V1q))))))
\end{aligned}$$