

thm\_2Earithmetic\_2EONE\_\_MOD  
(TMcajj4dLYgXHSvLCG2BqaiawbtzQqdBZZr)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{2}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega)^{ty\_2Enum\_2Enum} \tag{5}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega)^{\omega} \tag{6}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 V0n) V0n)$ .

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t) V2t)) V0t1)$ .

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21) V0t)$ .

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A^{27a} : \iota.(\lambda V0P \in (2^A)^{27a}).(ap V0P (ap (c\_2Emin\_2E\_40 V0P) V0P))$ .

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$ .

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\ & \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ & \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Enum\_2ESUC \\ & \quad V1m)) V2n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V1m) \\ & \quad V2n)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\ & \quad c\_2Enum\_2E0) V0n) = c\_2Enum\_2E0)) \wedge (\forall V1m \in ty\_2Enum\_2Enum. \\ & \quad (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\ & \quad ap c\_2Enum\_2ESUC V1m)) V2n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap ( \\ & \quad ap c\_2Earithmetic\_2E\_2A V1m) V2n)) V2n)))))) \end{aligned} \quad (11)$$

Assume the following.

$$((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0)) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1n)\ V2p)))) \Rightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V2p)))))) \quad (13)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ c\_2Enum\_2E0)\ V0n) = c\_2Enum\_2E0))) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EDIV\ c\_2Enum\_2E0)\ V0n) = c\_2Enum\_2E0))) \quad (15)$$

Assume the following.

$$(\forall V0y \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0y)) \wedge (\neg((ap\ c\_2Enum\_2ESUC\ V1x) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V1x)\ V0y)))\ V0y)))) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ c\_2Enum\_2ESUC\ V1x)\ V0y) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1x)\ V0y)))))) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (26)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) (ap c\_2Enum\_2ESUC V0n)))) \quad (27)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Rightarrow (\neg(V0m = V1n)))) \quad (28)$$

### Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))$$