

thm_2Arithmetic_2ONE__ONE__INV__IMAGE__BOUNDED
(TMZBT8LBpRJQan5jWRkaDpdosquRWnxxzRb)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P. 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 We define c_2Ebool_2E_21 to be $\lambda A_{-27} a : \iota. (\lambda V0P \in (2^{A_{-27}a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_{-27}a})$

Definition 6 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $\text{c.2Emin.2E.3D.3D.3E}$ to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow p \ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.(\lambda V3t4 \in 2.(\lambda V4t5 \in 2.(\lambda V5t6 \in 2.(\lambda V6t7 \in 2.(\lambda V7t8 \in 2.(\lambda V8t9 \in 2.(\lambda V9t10 \in 2.(\lambda V10t11 \in 2.(\lambda V11t12 \in 2.(\lambda V12t13 \in 2.(\lambda V13t14 \in 2.(\lambda V14t15 \in 2.(\lambda V15t16 \in 2.(\lambda V16t17 \in 2.(\lambda V17t18 \in 2.(\lambda V18t19 \in 2.(\lambda V19t20 \in 2.(\lambda V20t21 \in 2.(\lambda V21t22 \in 2.(\lambda V22t23 \in 2.(\lambda V23t24 \in 2.(\lambda V24t25 \in 2.(\lambda V25t26 \in 2.(\lambda V26t27 \in 2.(\lambda V27t28 \in 2.(\lambda V28t29 \in 2.(\lambda V29t30 \in 2.(\lambda V30t31 \in 2.(\lambda V31t32 \in 2.(\lambda V32t33 \in 2.(\lambda V33t34 \in 2.(\lambda V34t35 \in 2.(\lambda V35t36 \in 2.(\lambda V36t37 \in 2.(\lambda V37t38 \in 2.(\lambda V38t39 \in 2.(\lambda V39t40 \in 2.(\lambda V40t41 \in 2.(\lambda V41t42 \in 2.(\lambda V42t43 \in 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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty-2Enum-2Enum}) \quad (2)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega^{\omega}}) \quad (3)$$

Let $c_Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (4)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 13 We define $c_2Earithmetic_2EMAX$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 16 We define $c_2Ebool_2EONE_ONE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (ap\ (c_2Ebool_2EONE_ONE$

Let $c_2Enum_2EZZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZZERO_REP \in omega \quad (5)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZZERO_REP)$.

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \quad (6)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V0m))) \quad (7)$$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \wedge (\forall V1m \in ty_2Enum_2Enum. \\ & (\forall V2n \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ (ap\ c_2Enum_2ESUC\ V2n))) \Leftrightarrow ((V1m = (ap\ c_2Enum_2ESUC\ V2n)) \vee \\ & (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V2n))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V2p)\ (ap\ (ap\ c_2Earithmetic_2EMAX\ V1m)\ V0n))) \Leftrightarrow ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V2p)\ V1m)) \vee (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V2p)\ V0n)))))) \wedge ((\\ & p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2EMAX\ V1m)\ V0n))\ V2p)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V2p)) \wedge \\ & (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0n)\ V2p)))))) \end{aligned} \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. ((p \ V0t) \vee (\neg(p \ V0t)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge ((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}). ((p \ (ap \ (c.2Ebool.2EONE_ONE \ A.27a \\ & A.27b) \ V0f)) \Leftrightarrow (\forall V1x1 \in A.27a. (\forall V2x2 \in A.27a. (((ap \\ & V0f \ V1x1) = (ap \ V0f \ V2x2)) \Rightarrow (V1x1 = V2x2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p \ (ap \ V0P \ c.2Enum.2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p \ (ap \ V0P \ V1n)) \Rightarrow (p \ (ap \ V0P \ (ap \ c.2Enum.2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p \ (ap \ V0P \ V2n)))))) \end{aligned} \quad (17)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). ((p \ (ap \ (c.2Ebool.2EONE_ONE \\ & ty_2Enum_2Enum \ ty_2Enum_2Enum) \ V0f)) \Rightarrow (\forall V1b \in ty_2Enum_2Enum. \\ & (\exists V2a \in ty_2Enum_2Enum. (\forall V3x \in ty_2Enum_2Enum. (\\ & (p \ (ap \ (ap \ c.2Earithmetic.2E_3C_3D \ (ap \ V0f \ V3x)) \ V1b)) \Rightarrow (p \ (ap \ (ap \\ & c.2Earithmetic.2E_3C_3D \ V3x) \ V2a)))))) \end{aligned}$$