

thm\_2Earithmetic\_2ESUC\_\_MOD  
(TMKq7z3vdZQZNmnAhz4eGzQGrFLgHHwaCcT)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap c\_2Enum\_2EREP\_num (ap c\_2Enum\_2ESUC\_REP m)))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_2EZERO\_REP \in \omega \quad (6)$$

**Definition 14** We define  $c\_Enum\_2E0$  to be  $(ap\ c\_Enum\_2EABS\_num\ c\_Enum\_2EZERO\_REP)$ .

**Definition 15** We define  $c\_Earithmetic\_2EZERO$  to be  $c\_Enum\_2E0$ .

**Definition 16** We define  $c\_Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_Earithmetic$

**Definition 17** We define  $c\_Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 18** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 19** We define  $c\_Ebool\_2EBOUNDED$  to be  $(\lambda V0v \in 2. c\_Ebool\_2ET)$ .

Assume the following.

$$((ap\ c\_Earithmetic\_2ENUMERAL\ (ap\ c\_Earithmetic\_2EBIT1\ c\_Earithmetic\_2EZERO)) = (ap\ c\_Enum\_2ESUC\ c\_Enum\_2E0)) \quad (9)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg (c\_Enum\_2E0 = (ap\ c\_Enum\_2ESUC\ V0n)))) \quad (10)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_Earithmetic\_2E\_2B\ V0m)\ c\_Enum\_2E0) = V0m)) \quad (11)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \quad (12)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) (ap c\_2Enum\_2ESUC V1m)))))) \quad (13)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (V0m = V1n) \vee ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \vee (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m)))))) \quad (15)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (16)$$

Assume the following.

$$(\forall V0k \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EMOD V0k) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = c\_2Enum\_2E0)) \quad (17)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD V1k) V0n) = V1k)))) \quad (18)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0n)) \Rightarrow (\forall V1j \in ty\_2Enum\_2Enum. (\forall V2k \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2EMOD V1j) V0n)) (ap (ap c\_2Earithmetic\_2EMOD V2k) V0n))) V0n) = (ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2E\_2B V1j) V2k)) V0n)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V0n)) \Rightarrow (((ap (ap c\_2Earithmetic\_2EDIV V0n) V0n) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \wedge \\
& ((ap (ap c\_2Earithmetic\_2EMOD V0n) V0n) = c\_2Enum\_2E0))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V1n)) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad (ap (ap c\_2Earithmetic\_2EMOD V0m) V1n)) V1n))))))
\end{aligned} \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& \quad ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \quad (34)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap\ c\_2Ebool\_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n))) \quad (37)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ c\_2Enum\_2E0)))) \quad (38)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. ( \\ & \forall V2b \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ & V0n)) \Rightarrow (((ap (ap c\_2Earithmetic\_2EMOD (ap c\_2Enum\_2ESUC V1a)) \\ & V0n) = (ap (ap c\_2Earithmetic\_2EMOD (ap c\_2Enum\_2ESUC V2b)) V0n)) \Leftrightarrow \\ & ((ap (ap c\_2Earithmetic\_2EMOD V1a) V0n) = (ap (ap c\_2Earithmetic\_2EMOD \\ & V2b) V0n))))))) \end{aligned}$$