

thm\_2Earithmetic\_2EX\_LE\_DIV  
(TMKjqn6RHqP3NvdrJNK5ZLtY3XCH1frDyPN)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V 1x \in 2.V 1x)) (\lambda V 1x \in 2.V 1x)))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V 0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (c\_2Enum\_2EREP\_num m))$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{8}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n))$ .

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21\ 2))$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$ .

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A.\lambda y.y \in A))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^A)^{27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A)\ P))$ .

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$ .

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$ .

**Definition 17** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$ .

**Definition 18** We define  $c\_2Emarker\_2EAbbrev$  to be  $\lambda V0x \in 2.V0x$ .

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0m) = V0m) \wedge \\ & (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\ & ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n) = (ap \\ & (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)) \\ & V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ c\_2Enum\_2ESUC\ V1n)) = \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ & V0m)\ V1n)))))))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2A (ap \\
& (ap c\_2Earithmic\_2E\_2B V0m) V1n)) V2p) = (ap (ap c\_2Earithmic\_2E\_2B \\
& \quad (ap (ap c\_2Earithmic\_2E\_2A V0m) V2p)) (ap (ap c\_2Earithmic\_2E\_2A \\
& \quad \quad V1n) V2p))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmic\_2E\_2B V2p) V0m)) (ap (ap c\_2Earithmic\_2E\_2B \\
& \quad V2p) V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& \quad ap (ap c\_2Earithmic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V2p))) \Rightarrow (p (ap (ap \\
& \quad c\_2Eprim\_rec\_2E\_3C V0m) V2p))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmic\_2E\_2A V0m) V1n)) (ap (ap c\_2Earithmic\_2E\_2A \\
& \quad V2p) V1n))) \Leftrightarrow ((V1n = c\_2Enum\_2E0) \vee (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad \quad V0m) V2p))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmic\_2E\_2A V0m) V1n)) (ap (ap c\_2Earithmic\_2E\_2A \\
& \quad V2p) V1n))) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V1n)) \wedge \\
& \quad (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V2p))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum. ((V1k = (ap ( \\
& \quad ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A (ap (ap c\_2Earithmetic\_2EDIV \\
& \quad V1k) V0n)) V0n)) (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n))) \wedge (p (ap \\
& \quad (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EMOD V1k) V0n)) \\
& \quad V0n))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0x) V1y)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V0x) (ap (ap c\_2Earithmetic\_2E\_2B V1y) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{19}$$

Assume the following.

$$True \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& \quad (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& \quad (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& \quad A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg ( \\
& \quad p V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (28)$$

**Theorem 1**

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2z \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V2z)) \Rightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0x) (ap (ap c\_2Earithmetic\_2EDIV V1y) V2z))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2A V0x) V2z)) V1y)))))))))$$