

thm_2Earithmetic_2EX_LE_X_EXP
(TMFdgALX2rTA74TW6Z7TqVEaTmQoLTXGsRS)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 V0n) (ap c_2Earithmetic_2EBIT1 V0n))$.

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 V0P) (ap c_2Emin_2E_40 V0P))))$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2) V0t)$.

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 17 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in \\ & \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\ & \quad V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\ & \quad V1m) V2n))))))) \end{aligned} \quad (9)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum.(V0m = (ap c_2Enum_2ESUC V1n)))))) \quad (10)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (11)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee (p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V1n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) \\
& (ap (ap c_2Earithmetic_2E_2A V1n) V0m))) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee \\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1n))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2EEXP \\
& V0x) V1y))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0x)) \vee \\
& (V1y = c_2Enum_2E0))))
\end{aligned} \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{15}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Enum_2ESUC V0n) = c_2Enum_2E0))) \tag{20}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0n) V0n)))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Enum_2ESUC V0n)))) \quad (22)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1x) (ap (ap c_2Earithmetic_2EEXP V1x) V0n))))))$$